A Two-Person Game for Pricing Convertible Bonds

A firm issues a convertible bond. At each subsequent time, the bondholder must decide whether to keep the bond, thereby collecting *coupons*, or to *convert* it to stock. The bondholder wishes to choose a *conversion strategy* to maximize the bond value. Subject to some restrictions, a convertible bond can be *called* by the issuing firm, which presumably acts to maximize equity value and thus to minimize the bond value. This creates a two-person game, and we model the bond price as the value of this game. We show, however, that under our standing assumption (*dividends* are paid at a lower rate that the *money market rate*) this game reduces to one of two optimal stopping problems, and the relevant stopping problem can be determined a priori, i.e., without first solving the convertible bond pricing problem.

Because of *dividend payments*, the partial differential equation describing the pricing function becomes *nonlinear*. This means that our analysis involves a *fixed point* problem. We also prove that for large *time to maturity* the value of the convertible bond approaches the value of the *perpetual convertible bond*

The presentation is based on joint work with Steven E. Shreve.