Orientation of Rigid Bodies Freefalling in Newtonian and Non-Newtonian Fluids

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Abstract

This thesis deals with the subject of terminal orientations of rigid bodies, sedimenting in Newtonian and non-Newtonian liquids. It is a well established fact that homogeneous bodies of revolution around an axis (a) with fore-aft symmetry will orient themselves with respect to the direction of gravity (g) depending upon their shape and upon the nature of the fluid in which they are immersed. If, for instance, we are considering an ellipsoidal object falling in a Newtonian fluid such as water, then the body falls with a eventually becoming perpendicular to the direction of g. However, if the same body falls in a viscoelastic fluid where the inertial effects can be disregarded, then a will eventually become parallel to g. It has also been noted that long bodies falling in fluids with certain polymeric concentrations can take on angles between the horizontal and vertical orientations. These intermediate angles are referred to as *tilt angles*. The objective of this thesis is the explanation of this orientation phenomenon in different liquids.

Our approach to the problem has been three-fold, experimental, mathematical and also numerical. We perform several experiments on sedimentation of particles in a variety of viscoelastic and Newtonian liquids to verify and fill gaps in the previous experiments. A second set of experiments that we perform involves a modified flow chamber setup where the particle is fixed at the center of the chamber while water flows past it. We are able to replicate previous experiments at low and intermediate Re (Reynolds number), with both these experiments and also obtain new results concerning the steady state orientation of flat ended cylinders sedimenting in polymeric liquids.

The equations to describe the problem of freefall of a rigid body of arbitrary shape, in a liquid, are obtained from a frame attached to the body and is formulated for any general fluid model. In addition, since we are dealing with a fluid-solid problem, we also obtain the equations for the body in terms of the forces and torques imposed upon it, from the surrounding unbounded liquid. We establish well-posedness of the equations by showing the exitence and uniqueness of steady solutions to the problem of sedimentation in a Second order fluid, with $Re = 0$ and small We (Weissenberg number) using the Banach fixed point theorem. Considering various symmetries of the rigid body, we study the possible spin-free orientations that the body can assume, when the total torque acting on the body is in equilibrium.

In order to explain the terminal orientation assumed by the body, we consider the effect of torques imposed by different components of the liquid such as inertia, viscoelasticity and shear-thinning. The equilibrium resulting from the competition of the different torques should reveal the terminal angle. Guided by the fact that the orientation phenomenon is observed at very small Re and We, we formulate the torque equations at first order in these material parameters. The calculation is performed for four different liquid models, Newtownian, Power-law, Second order fluid and a modified-Second order model, where in the last model, the normal stress coefiicient α_2 depends upon the shear rate. The different orientation observations seen in experiments is well explained by these models. Finally, a simple quasi-steady stability argument is used to establish stability of the equilibrium states. For this final argument, we numerically evaluate the torque imposed by the individual components of the liquid upon a sedimenting prolate ellipsoid in an unbounded, three dimensional fluid domain surrounding the body.