SOLUTIONS

Math 259 **Winter 2009**

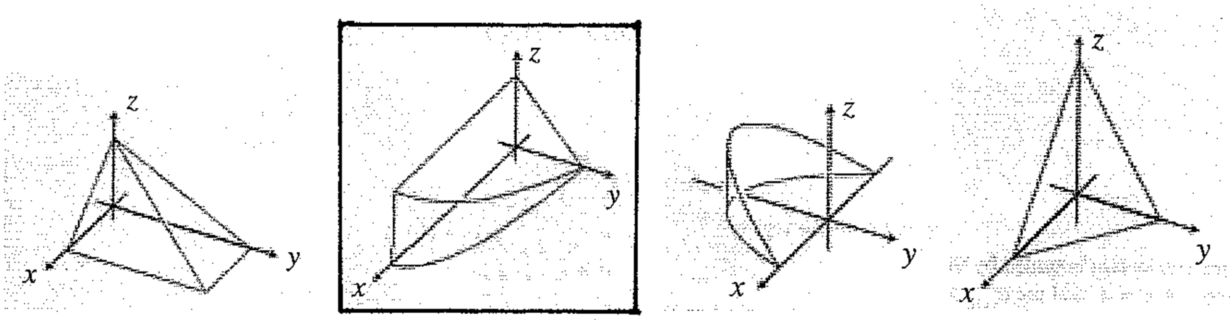
Quiz #9

1. In this problem you will always be concerned with the region in the first octant that is bounded by:

- $x = 0$
- $y = 0$
- $z = 0$
- $y + z = 2$
- $x = 4 - y^2$.

You should not use your calculator to evaluate integrals in this problem, apart from working out arithmetic and evaluating functions.

(a) **(1 point)** Which of the follow diagrams shows the region described above? **CIRCLE YOUR ANSWER.** If you circle more than one answer, you will receive zero credit.



(b) **(2 points)** Set up a **double** integral that will give the volume of the region described above.

$$\text{Volume} = \int_0^2 \int_0^{4-y^2} (2 - y)\, dx\, dy$$

(c) **(2 points)** Calculate the volume of the region described above. Show all of your work. If you give your answer as a decimal, include at least four (4) decimal places.

$$\text{Volume} = \int_0^2 \Big[(2 - y)\, x \Big]_0^{4-y^2} dy$$

$$= \int_0^2 (2 - y)(4 - y^2)\, dy$$

Additional space for your work is provided on the next page.

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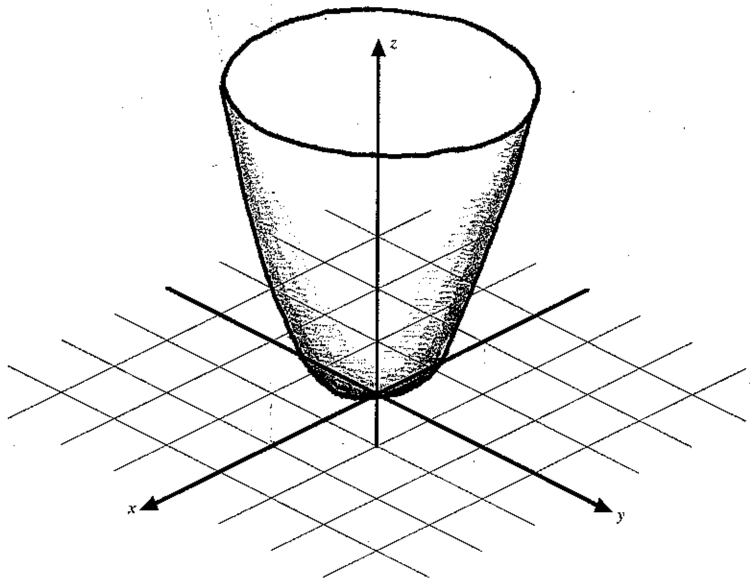
$$\text{Volume} = \int_0^2 (8 - 2y^2 - 4y + y^3)\,dy$$

$$= \left[8y - \frac{2}{3}y^3 - 2y^2 + \frac{1}{4}y^4 \right]_0^2$$

$$= \frac{20}{3} \text{ cubic units.}$$

SOLUTIONS

2. (a) **(1 point)** The surface S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$. Use the axes provided below to make an accurate sketch of the surface S.



(b) **(3 points)** Set up a **TRIPLE** integral in x, y, z coordinates that will give the volume enclosed by the surface S, the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$. You **do not** need to evaluate this integral.

There are many ways to write the triple integral, depending on how you order dx, dy, dz.

Here is one way:

$$\text{Volume} = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{0}^{x^2+y^2} 1 \cdot dz\, dy\, dx$$

SOLUTIONS

3. **(1 point)** Evaluate the following triple integral:

$$\int_0^1 \int_0^z \int_0^{x+z} 6 \cdot x \cdot z \cdot dy \cdot dx \cdot dz.$$

Show all of your work and clearly indicate your final answer. NO WORK = NO CREDIT.

You should **not use a calculator** in this part of the problem for anything besides arithmetic.

If you give your final answer as a decimal, include at least four (4) decimal places.

$$\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz$$

$$= \int_0^1 \int_0^z \left[6xyz \right]_0^{x+z} dx \, dz$$

$$= \int_0^1 \int_0^z 6xz(x+z) \, dx \, dz$$

$$= \int_0^1 \left[2x^3 z + 3x^2 z^2 \right]_0^z dz$$

$$= \int_0^1 5z^4 \, dz$$

$$= \left[z^5 \right]_0^1$$

$$= 1.$$