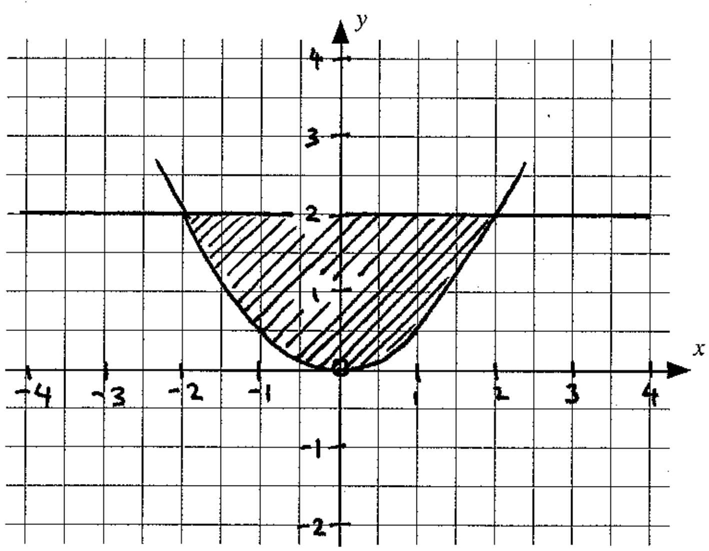
SOLUTIONS

Math 259 **Winter 2009**

Quiz #8

1. **(a)** **(1 point)** Use the axes given below to draw an accurate sketch of the region of the xy-plane that is bounded by the curves: $y = 2$ and $y = \frac{1}{2}x^2$.



(b) **(2 points)** Set up an integral that will give the amount of volume that lies below the plane:

$$3x + 2y - z = 0,$$

when the region of integration is the region described in Part (a).

$$\text{Volume} = \int_{-2}^{2} \int_{\frac{1}{2}x^2}^{2} (3x + 2y)\, dy\, dx$$

Also correct is:

$$\text{Volume} = \int_{0}^{2} \int_{-\sqrt{2y}}^{\sqrt{2y}} (3x + 2y)\, dx\, dy$$

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(c) **(2 points)** Evaluate the integral that you set up in Part (b) to calculate the volume beneath the plane. Show all of your work. You should **not use a calculator** in this part of the problem for anything besides simple arithmetic.

$$\text{Volume} = \int_{-2}^{2} \int_{\frac{1}{2}x^2}^{2} (3x + 2y)\, dy\, dx$$

$$= \int_{-2}^{2} \Big[3xy + y^2 \Big]_{\frac{1}{2}x^2}^{2} dx$$

$$= \int_{-2}^{2} \left(6x + 4 - \frac{3}{2}x^3 - \frac{1}{4}x^4\right) dx$$

$$= \left[3x^2 + 4x - \frac{3}{8}x^4 - \frac{1}{20}x^5 \right]_{-2}^{2}$$

$$= 12 + 8 - 6 - \frac{32}{20} - \left(12 - 8 - 6 + \frac{32}{20}\right)$$

$$= 16 - \frac{32}{20}$$

$$= \frac{188}{20}$$

(Evaluating the other integral will give the same result.)

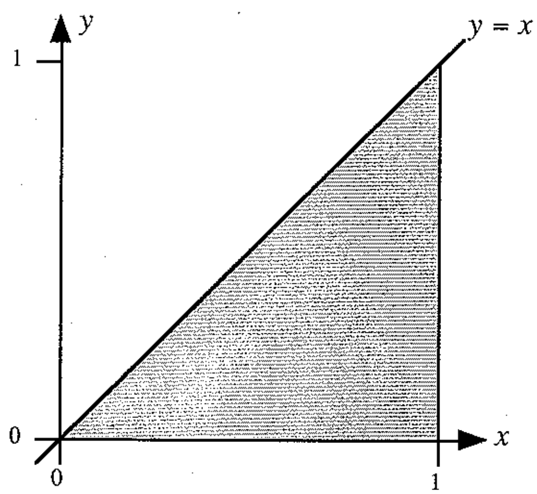
SOLUTIONS

2. **(3 points)** Evaluate the following double integral:

$$\int_0^1 \int_y^1 e^{-x^2}\, dx dy.$$

Show all of your work. NO WORK = NO CREDIT. You should **not use a calculator** in this part of the problem for anything besides simple arithmetic.

NOTE: The diagram given below shows the region of integration for this particular double integral (it is shaded).



It is not possible to find an anti-derivative for e^{-x^2} with respect to x that we can express in terms of elementary functions.

To evaluate the integral we will change the order of integration.

$$\int_0^1 \int_y^1 e^{-x^2}\, dx\, dy = \int_0^1 \int_0^x e^{-x^2}\, dy\, dx$$

$$= \int_0^1 \left[y \cdot e^{-x^2} \right]_0^x dx$$

$$= \int_0^1 x e^{-x^2}\, dx$$

$$= \left[-\tfrac{1}{2} e^{-x^2} \right]_0^1$$

$$= \tfrac{1}{2}(1 - e^{-1}).$$

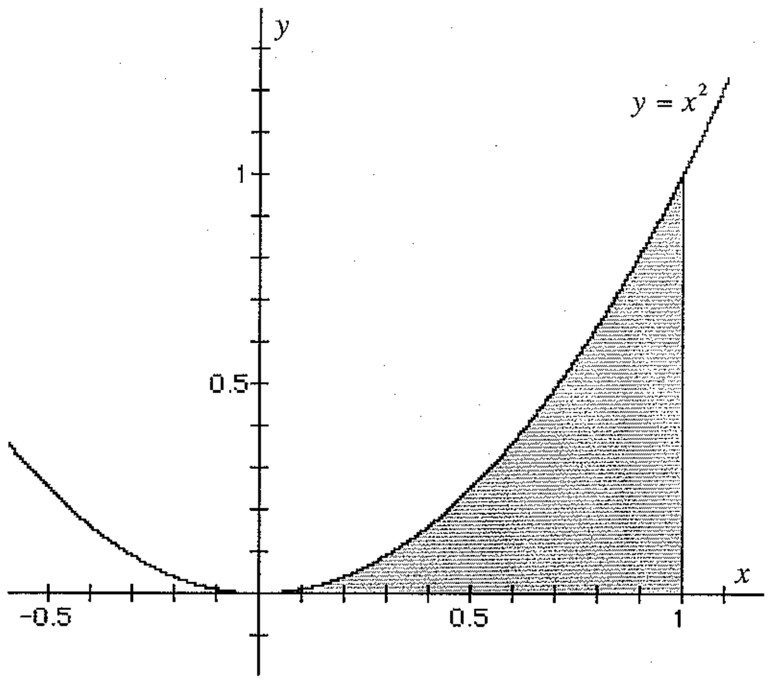
SOLUTIONS

3. **(2 points)** Evaluate the following double integral:

$$\int_0^1 \int_{\sqrt{y}}^1 \cos\left(x^3\right) dx\,dy.$$

Show all of your work. NO WORK = NO CREDIT. You should **not use a calculator** in this part of the problem for anything besides simple arithmetic.

NOTE: The diagram given below shows the region of integration for this particular double integral (it is shaded).



It is not possible to find an anti-derivative for $\cos(x^3)$ with respect to x that we can express in terms of elementary functions.

To evaluate this integral we will change the order of integration.

$$\int_0^1 \int_{\sqrt{y}}^1 \cos(x^3)\,dx\,dy = \int_0^1 \int_0^{x^2} \cos(x^3)\,dy\,dx$$

$$= \int_0^1 \Big[y \cdot \cos(x^3) \Big]_0^{x^2} dx$$

$$= \int_0^1 x^2 \cdot \cos(x^3) \cdot dx$$

$$= \Big[\tfrac{1}{3} \sin(x^3) \Big]_0^1$$

$$= \tfrac{1}{3} \sin(1).$$