Quiz #7

1.   **(5 points)** Find the maximum and minimum values attained by the function:

$$f(x, y, z) = xy + 2z$$

when it is subject to the constraint: $x^2 + y^2 + z^2 = 36$. Show all of your work. Record your final answers in the space provided on the next page.

$$f(x,y,z) = xy + 2z \qquad \nabla f = \langle y, x, 2 \rangle$$

$$g(x,y,z) = x^2 + y^2 + z^2 - 36 \qquad \nabla g = \langle 2x, 2y, 2z \rangle.$$

$$\nabla f = \lambda \cdot \nabla g$$

is equivalent to:

$$y = 2\lambda x \quad \cdots \; \textcircled{1}$$

$$x = 2\lambda y \quad \cdots \; \textcircled{2}$$

$$2 = 2\lambda z \quad \cdots \; \textcircled{3}$$

Substitute ② into ①: $y = 4\lambda^2 y \;\cdots\; \textcircled{4}$

From ④, either $(x, y, z) = (0, 0, 6)$ or

$$4\lambda^2 = 1 \quad \text{so} \quad \lambda = \pm \tfrac{1}{2}.$$

$\underline{\lambda = \tfrac{1}{2}} :$ $z = 2$

$y = x$ so that:

$$x^2 + x^2 + 2^2 = 36$$

ADDITIONAL SPACE TO SHOW YOUR WORK IS GIVEN ON THE NEXT PAGE.

Find the maximum and minimum values attained by the function:

$$f(x,y,z) = xy + 2z$$

when it is subject to the constraint: $x^2 + y^2 + z^2 = 36$. Show all of your work. Record your final answers in the space provided at the bottom of this page.

$$x = \pm 4$$

and $(x, y, z) = (\pm 4, \pm 4, 2)$.

$\underline{\lambda = -\frac{1}{2}:}$ $z = -2$

$y = -x$ so that:

$$x^2 + (-x)^2 + 2^2 = 36$$

$$x = \pm 4$$

and $(x, y, z) = (\pm 4, \mp 4, -2)$.

X	Y	Z	$f(x, y, z)$
0	0	6	12
4	4	2	20
-4	-4	2	20
4	-4	-2	-20
-4	4	-2	-20

MAXIMUM value attained by f(x, y, z) = _____ 20 _____

MINIMUM value attained by f(x, y, z) = _____ -20 _____

2. **(5 points)** A snake (on a plane) has found a warm metal plate to slither around on. The temperature of the plate (given in °C) at a point (x and y are both measured in meters) is given by the function:

$$T(x,y) = 4x^2 - 4xy + y^2.$$

The snake slithers around in a path that looks exactly like a circle of radius 5 meters centered on the origin. What are the highest and lowest temperatures encountered by the snake as it slithers around this circular path? Show your work and record your answers in the spaces provided on the next page.

$$T(x,y) = 4x^2 - 4xy + y^2 \qquad \nabla T = \langle 8x - 4y, -4x + 2y \rangle$$

$$g(x,y) = x^2 + y^2 - 25 \qquad \nabla g = \langle 2x, 2y \rangle$$

$$\nabla T = \lambda \nabla g$$

is equivalent to:

$$8x - 4y = 2\lambda x \quad \dots \text{①}$$

$$-4x + 2y = 2\lambda y \quad \dots \text{②}$$

Rearranging ① and ② gives:

$$(4 - \lambda)x - 2y = 0 \quad \dots \text{③}$$

$$-2x + (1 - \lambda)y = 0 \quad \dots \text{④}$$

Solving this system of linear equations for x, y:

$$(\lambda^2 - 5\lambda)x = 0 \quad \dots \text{⑤}$$

$$(\lambda^2 - 5\lambda)y = 0 \quad \dots \text{⑥}$$

Solving these equations will give the x and y coordinates of the points that need to be checked.

ADDITIONAL SPACE TO SHOW YOUR WORK IS GIVEN ON THE NEXT PAGE.

SOLUTIONS

Solving Equation ⑤:

Either $x = 0$, $\lambda = 0$ or $\lambda = 5$.

$x = 0$: $\qquad (0)^2 + y^2 = 25 \qquad$ so $\quad y = \pm 5$.

$\lambda = 0$: $\qquad$ From ③: $\qquad 4x - 2y = 0$
$$2x = y$$
$$x^2 + (2x)^2 = 25 \qquad \text{so} \qquad x = \pm\sqrt{5}$$

$\lambda = 5$: $\qquad$ From ③: $\qquad -x - 2y = 0$
$$-2y = x$$
$$(-2y)^2 + y^2 = 25 \qquad \text{so} \qquad y = \pm\sqrt{5}$$

Solving Equation ⑥:

Either $y = 0$, $\lambda = 0$ or $\lambda = 5$.

$y = 0$: $\qquad x^2 + (0)^2 = 25 \qquad$ so $\quad x = \pm 5$.

$\lambda = 0$ and $\lambda = 5$: Same as above.

Checking Points:

x	y	$T(x,y)$
0	± 5	25
± 5	0	100
$\sqrt{5}$	$2\sqrt{5}$	0
$-\sqrt{5}$	$-2\sqrt{5}$	0
$-2\sqrt{5}$	$\sqrt{5}$	125
$2\sqrt{5}$	$-\sqrt{5}$	125

SOLUTIONS

A snake (on a plane) has found a warm metal plate to slither around on. The temperature of the plate (given in °C) at a point (x and y are both measured in meters) is given by the function:

$$T(x,y) = 4x^2 - 4xy + y^2.$$

The snake slithers around in a path that looks exactly like a circle of radius 5 meters centered on the origin. What are the highest and lowest temperatures encountered by the snake as it slithers around this circular path? Show your work and record your answers in the spaces provided below.

THE **HIGHEST** TEMPERATURE ENCOUNTERED BY THIS %+$*ſ%!@ING SNAKE WAS:

125 °C

THE **LOWEST** TEMPERATURE ENCOUNTERED BY THIS %+$*ſ%!@ING SNAKE WAS:

0 °C