SOLUTIONS

Quiz #6

1. In each part of this problem you are given an equation that you should interpret as the equation of a surface in 3D. For each equation:

(i) Identify the type of surface (plane, sphere, elliptic paraboloid, etc.).

(ii) Locate the x, y and z intercepts (if any).

(iii) Use the axes provided to draw an accurate sketch of the surface.

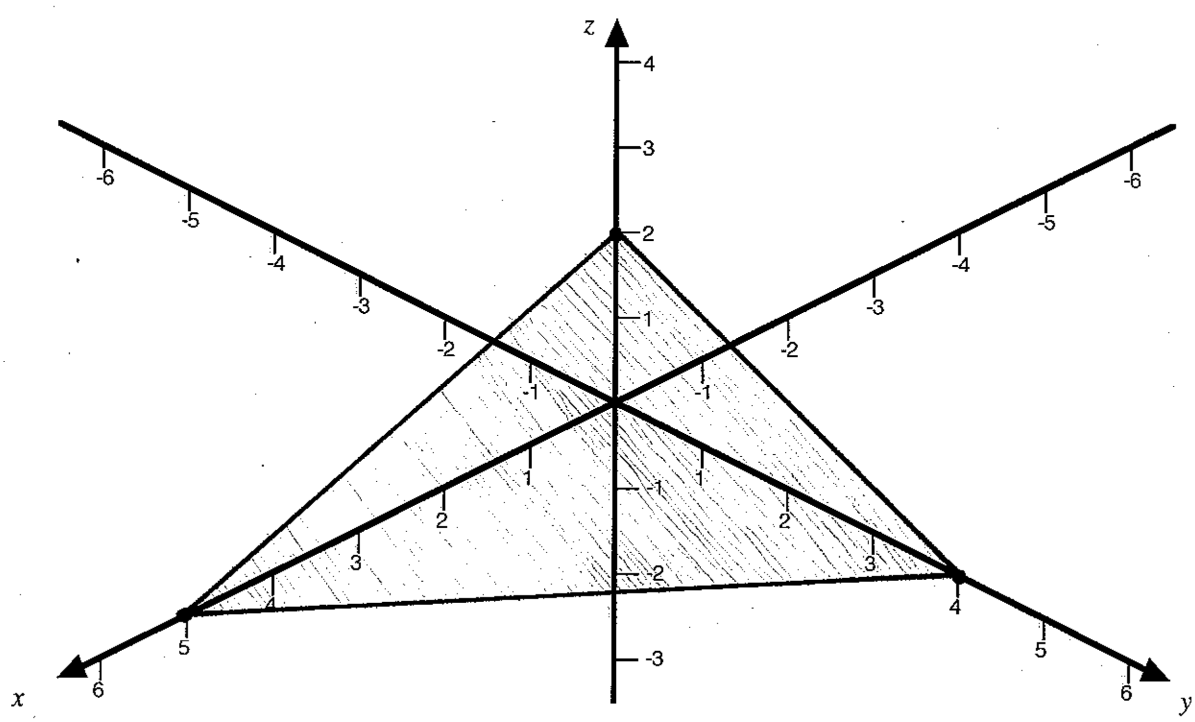
(a) **(3 points)** $4x + 5y + 10\,z = 20.$

The surface is a plane.

x-intercept: $(5, 0, 0)$ (set y & $z = 0$).

y-intercept: $(0, 4, 0)$ (set x & $z = 0$).

z-intercept: $(0, 0, 2)$ (set x & $y = 0$).



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(iii) Use the axes provided to draw an accurate sketch of the surface.

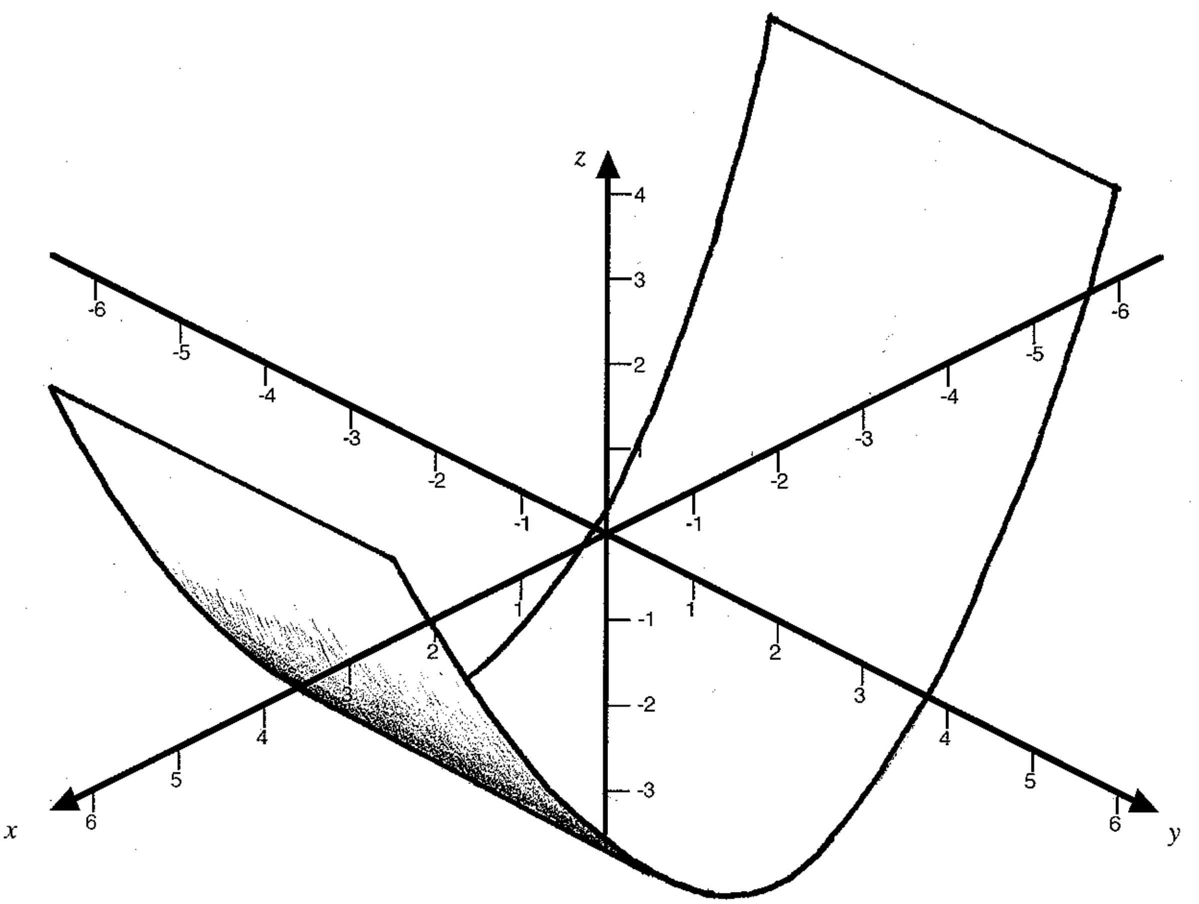
(b) **(3 points)** $x^2 = z + 3$.

The surface is a cylinder that extends along the y-axis.

x-intercepts: $(\sqrt{3}, 0, 0)$ and $(-\sqrt{3}, 0, 0)$

y-intercepts: None

z-intercept: $(0, 0, -3)$



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(i) Identify the type of surface (plane, sphere, elliptic paraboloid, etc.).

(ii) Locate the x, y and z intercepts (if any).

(iii) Use the axes provided to draw an accurate sketch of the surface.

(c) **(3 points)** $x^2 + 4y^2 - z^2 = 4.$

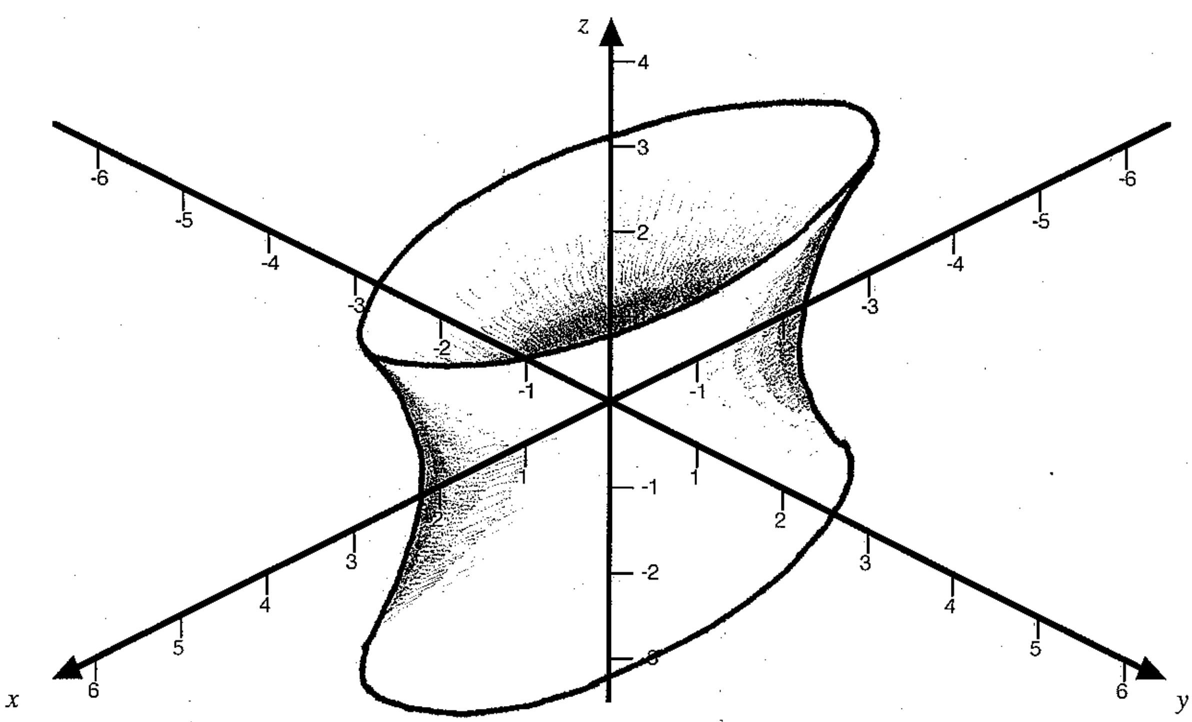
The surface is a hyperboloid of one sheet.

The intercepts are:

x-intercepts: $(-2, 0, 0)$ and $(2, 0, 0)$

y-intercepts: $(0, -1, 0)$ and $(0, 1, 0)$.

z-intercepts: None.



SOLUTIONS.

2. **(1 points)** Does the limit:

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{\sqrt{x^2+y^2}}$$

exist or not? Either calculate the value of the limit (showing your work – no work means no credit) or demonstrate that the limit does not exist.

<u>Claim:</u>

$$\lim_{(x,y)\to(0,0)} \frac{x\cdot y}{\sqrt{x^2+y^2}} = 0.$$

<u>Proof:</u> Let $\varepsilon > 0$ be given. Note that:

$$(x-y)^2 \geqslant 0$$

$$x^2 - 2xy + y^2 \geqslant 0$$

$$x^2 + y^2 \geqslant 2xy$$

So that: $|x\cdot y| \leq \frac{1}{2}(x^2+y^2)$.

Let $\delta = 2\varepsilon$ and assume $\sqrt{x^2+y^2} < \delta = 2\varepsilon$.

Then:

$$\left|\frac{x\cdot y}{\sqrt{x^2+y^2}}\right| = \frac{|x\cdot y|}{\sqrt{x^2+y^2}} \leq \frac{\frac{1}{2}(x^2+y^2)}{\sqrt{x^2+y^2}} = \frac{1}{2}\sqrt{x^2+y^2}$$

$$< \frac{1}{2}\delta = \frac{1}{2}(2\varepsilon) = \varepsilon.$$

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