SOLUTIONS

**Math 259** **Winter 2009**

## Quiz #4

**1.** In this problem you will study the conic section defined by the polar equation:

$$r = \frac{4}{2+\cos(\theta)} = \frac{2}{1+\frac{1}{2}\cos(\theta)} = \frac{\frac{1}{2}\cdot 4}{1+\frac{1}{2}\cos(\theta)}$$

In each case, clearly indicate your final answer (e.g. by circling it).

**(a)** **(1 point)** Find the eccentricity of the conic section.

Eccentricity $= e = \frac{1}{2}$

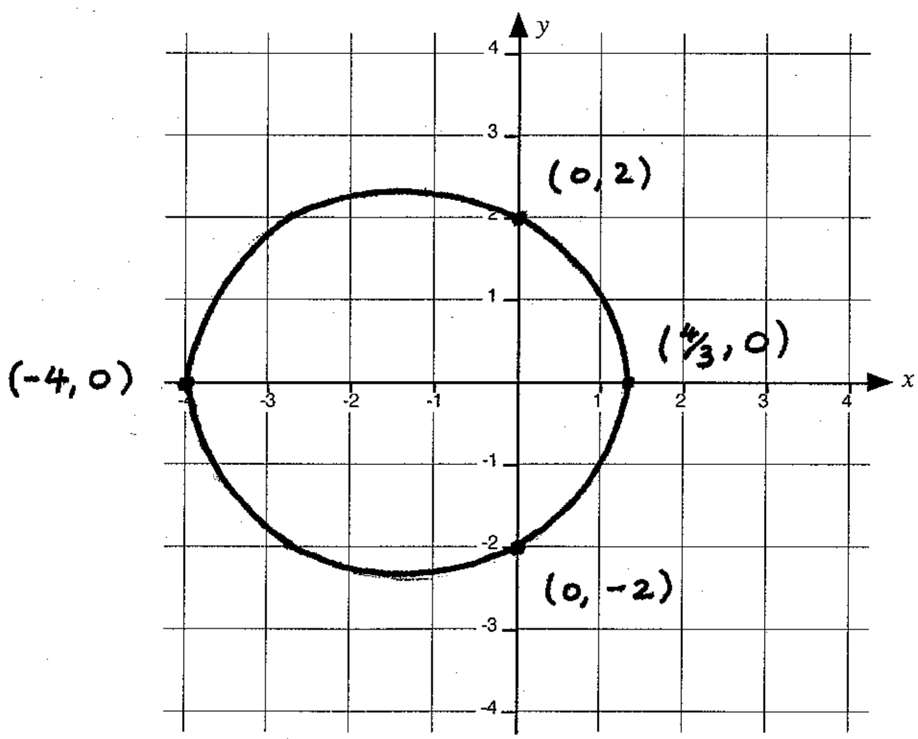
**(b)** **(1 point)** Classify the curve (ellipse, parabola or hyperbola).

As $e<1$, the curve is an ellipse.

**(c)** **(1 point)** Find the equation of the directrix.

$x = 4$. (Vertical because $r = \frac{ed}{1+e\cdot\cos(\theta)}$ format.)

**(d)** **(2 points)** Use the axes given below to draw a graph of the conic section in the *xy*-**plane.**



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**2.** **(2 points)** Find a unit vector with the same direction as the vector:

$$-2\vec{i} - \vec{j} + 5\vec{k}.$$

Clearly indicate your final answer.

The length of $\vec{v} = -2\vec{i} - \vec{j} + 5\vec{k}$ is:

$$|\vec{v}| = \sqrt{(-2)^2 + (-1)^2 + 5^2} = \sqrt{30}.$$

A unit vector with the same direction as $\vec{v}$ is:

$$\vec{u} = \frac{-2}{\sqrt{30}}\vec{i} - \frac{1}{\sqrt{30}}\vec{j} + \frac{5}{\sqrt{30}}\vec{k}.$$

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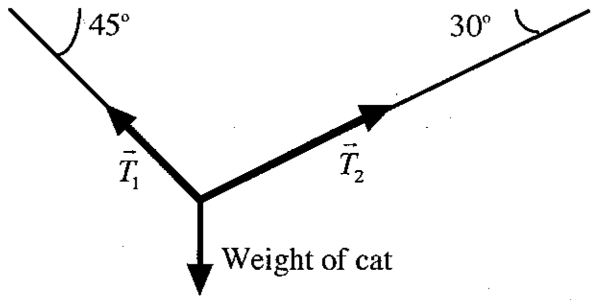
**3.** **(3 points)** While attempting the perilous maneuver of walking across a rope, a cat slipped and almost plummeted to its death. Fortunately the cat managed to grab the rope before it fell. Work out the tension (i.e. the vector $\vec{T}_1$, not just the magnitude of the vector) in the rope that is to the left of the cat. You can assume that the cat has a mass of 1 kg and that the acceleration due to gravity is 9.8 ms$^{-2}$.

Show all of your work and clearly indicate your final answer. The diagram shown below may be helpful, although it is **not drawn to scale.**

$T_1$, $T_2$ denote magnitudes of $\vec{T}_1$ and $\vec{T}_2$.




$- \longrightarrow +$

$\uparrow^{+}_{-}$

Sign convention.

Horizontal Components: $\quad -T_1 \cdot \cos\left(\frac{\pi}{4}\right) + T_2 \cdot \cos\left(\frac{\pi}{6}\right) = 0$

Vertical Components: $\quad T_1 \cdot \sin\left(\frac{\pi}{4}\right) + T_2 \cdot \sin\left(\frac{\pi}{6}\right) - 9.8 = 0$

Solving the system of linear equations for $T_1$ and $T_2$ gives:

$$T_2 = \frac{9.8}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{19.6}{1 + \sqrt{3}} \text{ N}$$

$$T_1 = \frac{\sqrt{6}}{2} T_2 = \frac{19.6\sqrt{6}}{2 + 2\sqrt{3}} \text{ N.}$$

The two vectors are:

$$\boxed{\begin{aligned}\vec{T}_1 &= -T_1 \cdot \cos\left(\tfrac{\pi}{4}\right)\vec{i} + T_1 \cdot \cos\left(\tfrac{\pi}{4}\right)\cdot\vec{j} \\ &= \frac{-19.6\sqrt{3}}{2 + 2\sqrt{3}}\vec{i} + \frac{19.6\sqrt{3}}{2 + 2\sqrt{3}}\vec{j}\end{aligned}}$$

$$\vec{T}_2 = T_2 \cdot \cos\left(\frac{\pi}{6}\right)\vec{i} + T_2 \sin\left(\frac{\pi}{6}\right)\vec{j}$$

$$= \frac{19.6\sqrt{3}}{2 + 2\sqrt{3}}\vec{i} + \frac{19.6}{2 + 2\sqrt{3}}\vec{j}$$

(Not required for answer but a good way to check $\vec{T}_1$ calculation.)