SOLUTIONS

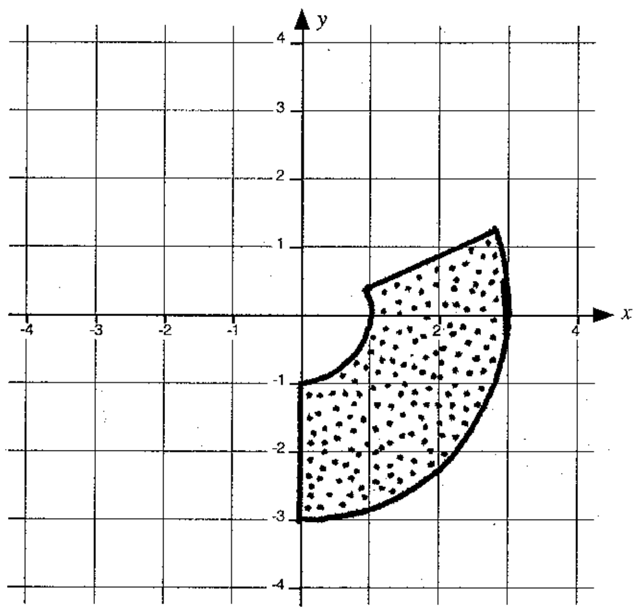
Math 259 **Winter 2009**

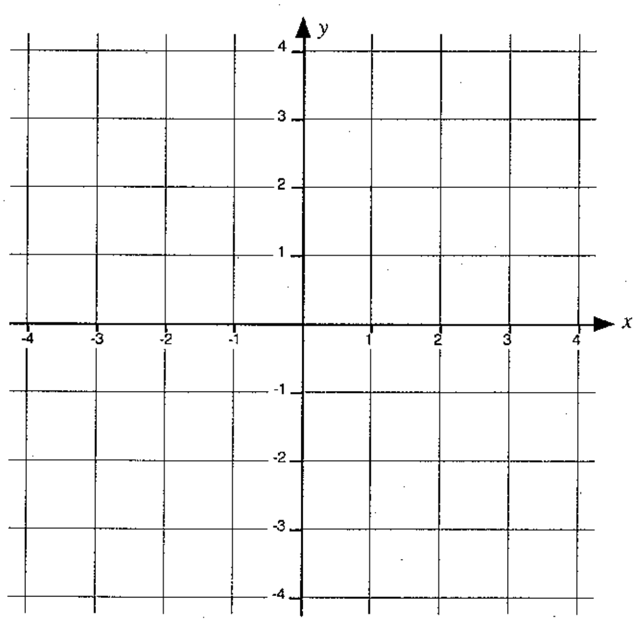
Quiz #3

1. **(2 points)** Use the axes provided below to sketch the region in the *xy*-plane consisting of points whose polar coordinates satisfy:

$$1 \le r \le 3 \text{ and } \frac{-\pi}{2} \le \theta \le \frac{\pi}{6}.$$

Two sets of axes are provided below in case you change your mind. Clearly indicate which one is your final answer – otherwise the grader will give you zero points.





SOLUTIONS

2. Consider the curve in the xy-plane defined by the polar equation:

$$r = \tan(\theta) \cdot \csc(\theta).$$

(a) **(2 points)** Find a Cartesian equation (i.e. one that involves only x, y and constants) for the curve. Show your work – no work = no credit.

Note that $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ and $\csc(\theta) = \dfrac{1}{\sin(\theta)}$.

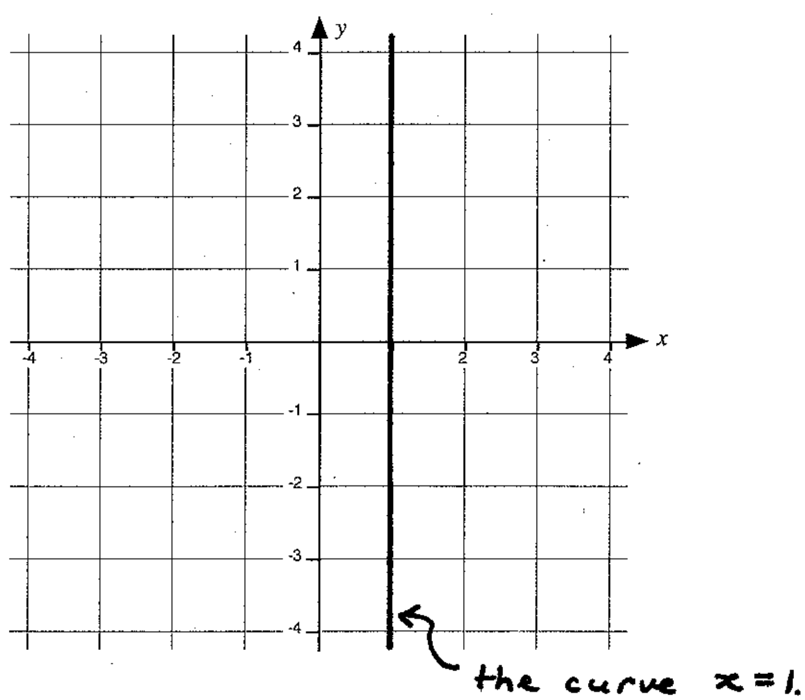
So, $r = \tan(\theta) \cdot \csc(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} \cdot \dfrac{1}{\sin(\theta)}$

So, $r = \dfrac{1}{\cos(\theta)}$ or $r \cdot \cos(\theta) = 1.$

Now, $x = r \cdot \cos(\theta)$ so the curve defined by the polar equation is:

$$x = 1.$$

(b) **(1 point)** Use the axes provided below to sketch the curve defined by the polar equation $r = \tan(\theta) \cdot \csc(\theta)$ in the xy-plane.

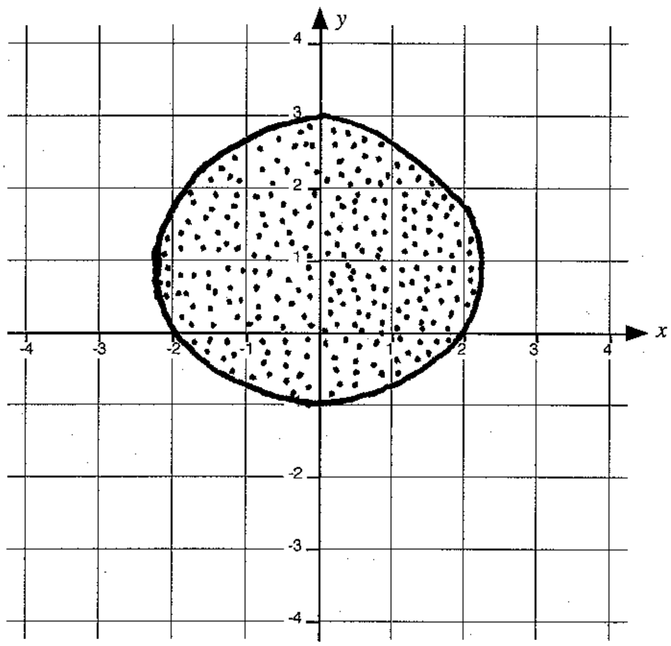


SOLUTIONS

3. Consider the curve in the xy-plane enclosed by the polar equation:

$$r = 2 + \sin(\theta).$$

(a) **(1 point)** Use the axes given below to sketch the area enclosed by the polar equation.



(b) **(2 points)** Set up an integral that will give the area enclosed by the polar equation.

$$\text{Area} = \frac{1}{2}\int_0^{2\pi} (2 + \sin(\theta))^2 \, d\theta$$

(c) **(2 points)** Evaluate your integral from Part (b). It may be helpful to know that $\sin^2(x) = \frac{1}{2}\cdot(1-\cos(2x))$.

You should not use your calculator on this problem for anything except simple arithmetic. If you need to find any antiderivatives, you should show your work. Finding antiderivatives on a calculator is not acceptable.

$$\frac{1}{2}\int_0^{2\pi} (2+\sin(\theta))^2 \, d\theta = \frac{1}{2}\int_0^{2\pi} (4 + 4\sin(\theta) + \sin^2(\theta)) \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \left(4 + 4\sin(\theta) + \frac{1}{2} - \frac{1}{2}\cos(2\theta)\right) d\theta$$

$$= \frac{1}{2}\left[\frac{9}{2}\theta - 4\cos(\theta) - \frac{1}{4}\sin(2\theta)\right]_0^{2\pi}$$

$$= \frac{9}{2}\pi$$