Handout 5: In-class Review for Exam 3

The topics covered by Exam 3 in the course include the following:

- Lagrange Multipliers.
- Lagrange Multipliers with two constraints.
- Word problems that can be solved using Lagrange Multipliers.
- Double Riemann sums.
- The Midpoint Rule for double Riemann sums (no error estimates).
- Setting up double integrals.
- Changing the order of integration in a double integral.
- Evaluating double integrals.
- Applications of double integrals to chemistry and physics (e.g. volume, mass, center of mass).
- Polar coordinates.
- Converting double integrals from Cartesian to polar coordinates.
- Setting up double integrals in polar coordinates.
- Evaluating double integrals in polar coordinates.
- Setting up triple integrals.
- Changing the order of integration for a triple integral.
- Evaluating triple integrals.
- Applications of triple integrals (e.g. calculating volume and mass).
- Setting up and evaluating triple integrals in cylindrical/polar coordinates.
- Setting up and evaluating triple integrals in spherical coordinates.
- Drawing two-dimensional vector fields.
- Associating formulas for two-dimensional vector fields with visual representations.
- Determining whether a vector field is conservative or not.
- Finding potential functions for conservative vector fields.
- Setting up parametric equations for curves in two dimensions.
- Setting up and evaluating (directly) line integrals of functions in two dimensions.
- Setting up and evaluating (directly) line integrals of vector fields in two and three dimensions.
- Calculating line integrals of conservative vector fields using the Fundamental Theorem.
- Calculating line integrals in two dimensions using Green's Theorem.

1. Find the volume of the solid bounded by the surface:

$$z = x\sqrt{x^2 + y}$$

and the planes: $x = 0$, $x = 1$, $y = 0$, $y = 1$, and $z = 0$.

2. The following integrals are given with the function $f(x, y)$ unspecified. Suppose that the function $f(x, y)$ was specified and the formula for $f(x, y)$ is one where the integral would be a lot easier to evaluate if the dx and dy were swapped. Write down what the new integrals would be (with the order of dx and dy swapped).

(a) $$\int_0^1 \int_0^x f(x, y)\,dy\,dx$$

(b) $$\int_1^2 \int_0^{\ln(x)} f(x, y)\,dy\,dx$$

(c) $$\int_0^4 \int_{y/2}^2 f(x, y)\,dx\,dy$$

3. Evaluate the line integral,

$$\int_C y^2\,dx + x\,dy$$

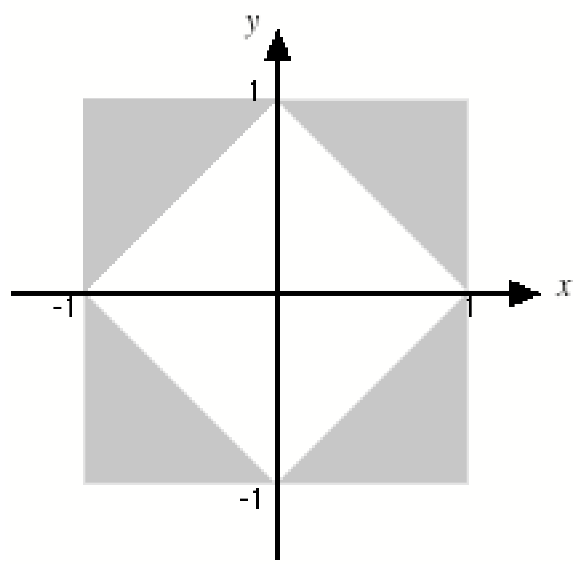
(a) When C is the line segment from (-5, -3) to (0, 2).

(b) When C is the part of the parabola from $x = 4 - y^2$ from (-5, -3) to (0, 2).

(c) How could you tell that the two integrals would yield different values without evaluating the integrals?

4. The region of integration is the shaded region D shown in the diagram below. Find the value of the integral,

$$\iint_D x^2 \, dx dy \ .$$



5. Decide whether or not the following vector fields are conservative. If any are conservative, find a function f so that $\nabla f = \boldsymbol{F}$.

(a) $\boldsymbol{F}(x, y, z) = yz\boldsymbol{i} + xz\boldsymbol{j} + xy\boldsymbol{k}$.

(b) $\boldsymbol{F}(x, y, z) = 2xy\boldsymbol{i} + (x^2 + 2yz)\boldsymbol{j} + y^2\boldsymbol{k}$.

(c) $\boldsymbol{F}(x, y, z) = e^x\boldsymbol{i} + e^y\boldsymbol{j} + e^z\boldsymbol{k}$.

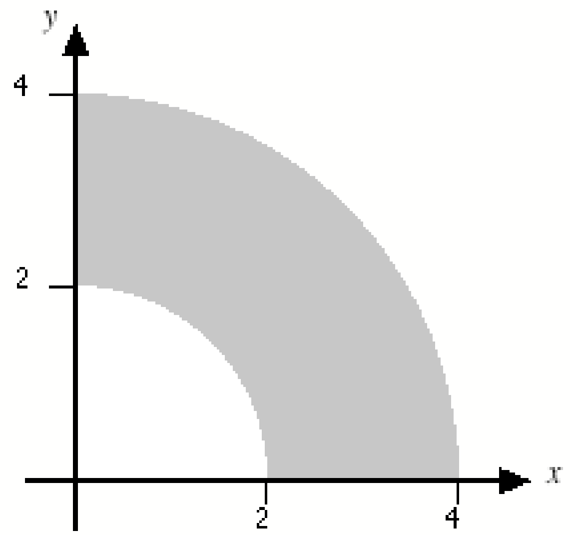
6. Evaluate the line integral, where C is the given curve.

(a) $\int_C y\,ds$, where C: $x = t^2$, $y = t$, $0 \le t \le 2$.

(b) $\int_C xy^4\,ds$, where C is the right hand side of the circle $x^2 + y^2 = 16$.

(c) $\int_C \left(xy + \ln(x)\right)dy$, where C is the arc of the parabola $y = x^2$ from $(1, 1)$ to $(3, 9)$.

7. **(a)** Sketch the solid that has a height given by: $z = xy$, and a base given by the shaded region in the diagram below.



(b) Set up in integral (in terms of x, y, dx, dy) that gives the volume of the solid.

(c) Evaluate the integral to find the volume.

8. Find the global maximum and global minimum values of the function:

$$f(x,y,z) = 3x - y - 3z$$

subject to the constraints:

$$x + y - z = 0 \qquad \text{and} \qquad x^2 + 2z^2 = 1.$$

9. Three alleles (alternative version of a gene) A, B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO) and AB. The Hardy-Weinberg law states that the proportion of individuals in a population who carry two different alleles is:

$$P = 2pq + 2pr + 2rq$$

where p, q, and r are the proportions of A, B, and O in the population. Use the fact that $p + q + r = 1$ to show that P is at most two thirds.

10. Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

(a) $\int_C e^y\,dx + 2xe^y\,dy$, where C is the square with sides $x = 0$, $x = 1$, $y = 0$ and $y = 1$.

(b) $\int_C \left(y + e^{\sqrt{x}}\right)dx + \left(2x + \cos\left(y^2\right)\right)dy$, where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

(c) $\int_C y^3\,dx + x^3\,dy$, where C is the circle $x^2 + y^2 = 4$.