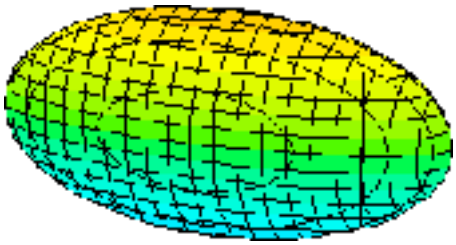
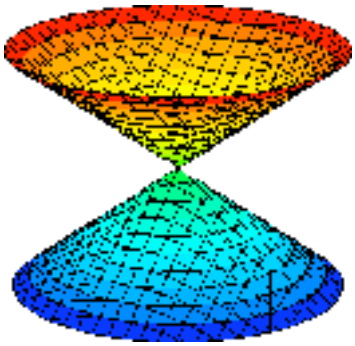
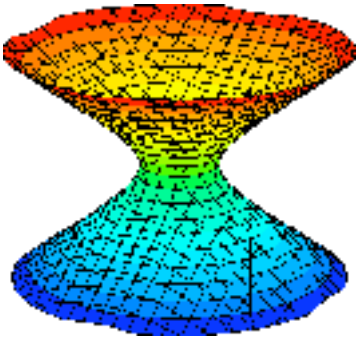
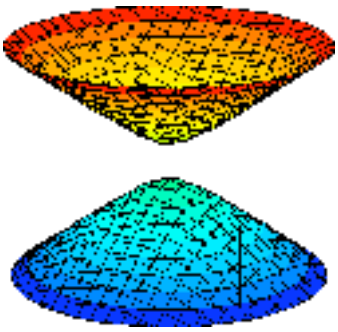
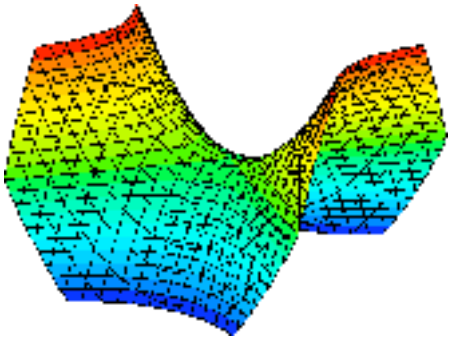
Handout 3: Classifying Quadric Surfaces

The key to classifying conic sections (in polar coordinates) was the eccentricity. It is possible to calculate similar numbers that can be used to classify quadric surfaces but this is much more complicated than eccentricity. We will classify quadric surfaces by the structure of the equation for the surface. The formula for each quadric surface is summarized in the table given below (were we assume that a, b and c are all non-zero constants).

Surface	Graph	Equation	Comments
Ellipsoid		$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$	Sphere is the case when $a = b = c$. The point (x_0, y_0, z_0) is at the very center of the ellipsoid.
Cone		$\frac{(z-z_0)^2}{c^2}=\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}$	The point (x_0, y_0, z_0) is the point where the two parts of the cone meet. If either x or y appears as the subject of the equation, then the cone opens along that axis instead.
Elliptic paraboloid		$\frac{z-z_0}{c}=\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}$	One of the variables will be raised to the first power. This gives the axis that the paraboloid opens along. The case $c > 0$ is illustrated here. The point (x_0, y_0, z_0) is the lowest point on the paraboloid.

Hyperboloid of one sheet		$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}-\frac{(z-z_0)^2}{c^2}=1$	Only one of the terms will be subtracted. This gives the axis along which the hyperboloid opens. The point (x_0, y_0, z_0) is at the center of the thinnest part of the surface
Hyperboloid of two sheets		$-\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}+\frac{(z-z_0)^2}{c^2}=1$	Only one of the terms will be added. This gives the axis along which the hyperboloid opens. The point (x_0, y_0, z_0) is at the halfway point between the sheets.
Hyperbolic paraboloid		$\frac{z-z_0}{c}=\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}$	This surface has a saddle shape like a Pringles potato chip. It is also possible for x to be subtracted and y added. The case $c < 0$ is illustrated here.