Handout 2: In-Class Review for Exam 1

The topics covered by Exam 1 in the course include the following:

- Parametric curves.
- Finding formulas for parametric curves.
- Drawing graphs of curves defined by parametric equations.
- Finding tangent lines to curves defined by parametric equations.
- Finding the area beneath (between the curve and the x-axis) a parametric curve.
- Finding the arc length of a parametric curve.
- Polar coordinates for the xy-plane.
- Identifying regions of the xy-plane described by polar coordinates.
- Converting Cartesian equations to polar equations.
- Converting polar equations to Cartesian equations.
- Sketching curves in the xy-plane defined by polar equations.
- Finding formulas for tangent lines to curves defined by polar equations.
- Finding areas enclosed by polar curves.
- Finding arc lengths of curves defined by polar equations.
- Conic sections in Cartesian and polar coordinates.
- Sketching conic sections defined by polar equations. Identifying eccentricity, directrix, etc. from a polar equation. Classifying conic sections using eccentricity.
- Equations of lines, planes and spheres in 3D.
- Combining vectors. Magnitude of a vector. Unit vectors.
- Applications of vectors in physics.
- Dot product of vectors. Angle between vectors. Orthogonality. Vector projections.
- Cross product of vectors. Geometry of the cross product. Cross product and areas.
- Calculating volumes with the scalar triple product.
- Finding equations for lines and planes in 3D using the cross product.
- Distances from points to lines and planes, and from lines to planes.
- Symmetric equations.

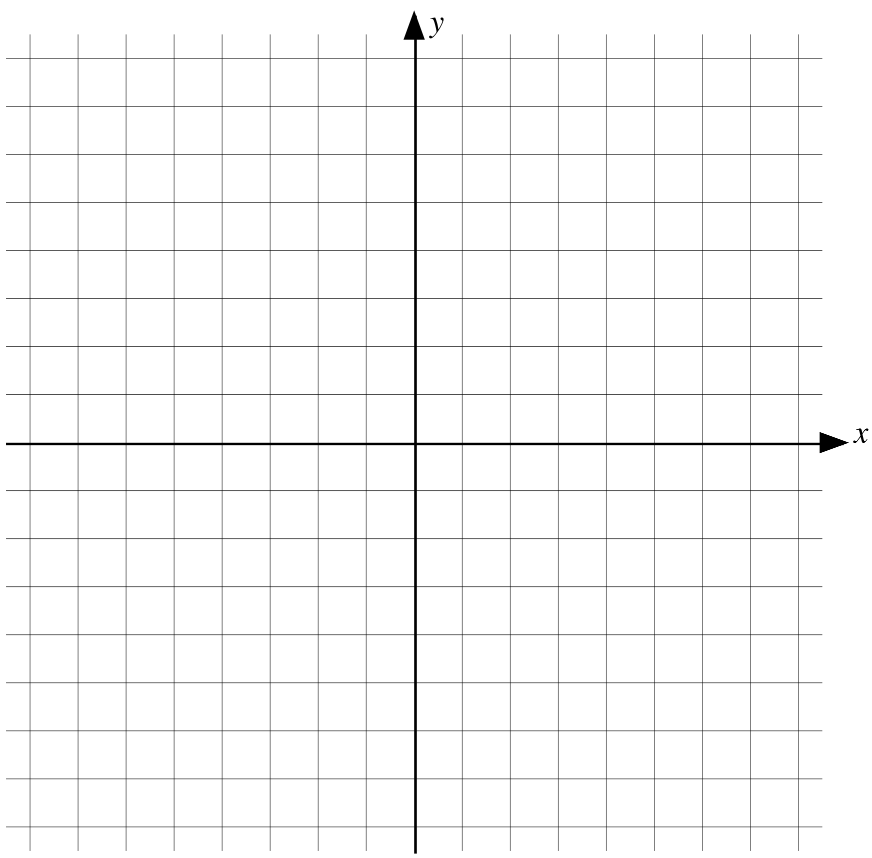
1. For each of the curves defined below:

(i) $$r = \frac{9}{6 + 2\cdot\cos(\theta)}$$ (ii) $$r = \frac{9}{2\cdot\cos(\theta)},$$

(a) Determine the eccentricity.

(b) Identify the type of curve.

(c) Sketch an accurate graph of the curve using the axes provided.


y
x

2. Find parametric equations for each of the following lines described below.

(a) The line that passes through the point $(-2, 2, 4)$ and is perpendicular to the plane $2x - y + 5z = 12$.

(b) The line that passes through the two points $(4, -1, 2)$ and $(1, 1, 5)$.

(c) The line that is formed by the intersection of the two planes $x + y - z = 1$ and $2x - 3y + 4z = 5$.

3. Find the length of each of the following curves.

(a) $x(t) = 3t^2$ and $y(t) = 2t^3$ where $0 \le t \le 2$.

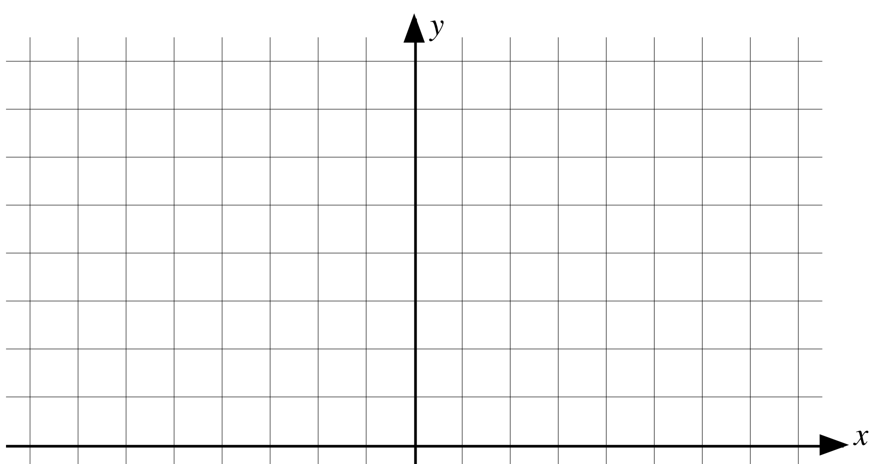
(b) $r = \dfrac{1}{\theta}$ where $\pi \le \theta \le 2\pi$.

(c) $r = \sin^3\left(\frac{\theta}{3}\right)$ where $0 \le \theta \le \pi$.

4. In this problem you may assume that a is a positive constant. Find the coordinates (x and y) of the points where the curve defined by:

$$x(t) = 2a \cdot \cos(t) - a \cdot \cos(2t) \qquad \text{and} \qquad y(t) = 2a \cdot \sin(t) - a \cdot \sin(2t)$$

has **(a)** horizontal, and **(b)** vertical tangent lines. Once you have identified these points, use the axes provided (see next page) to sketch the curve.


y
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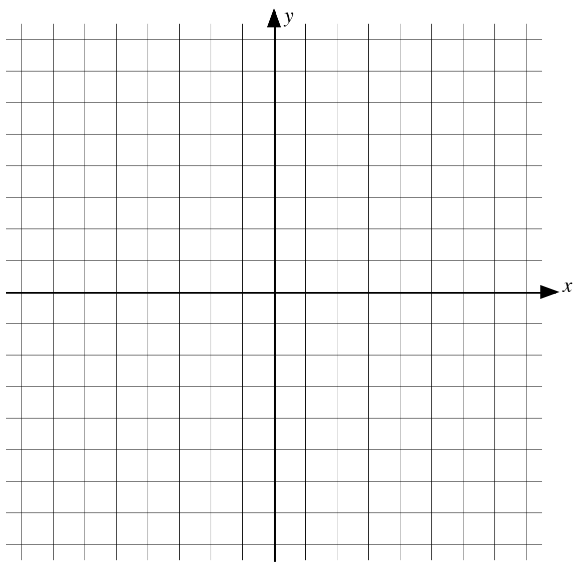
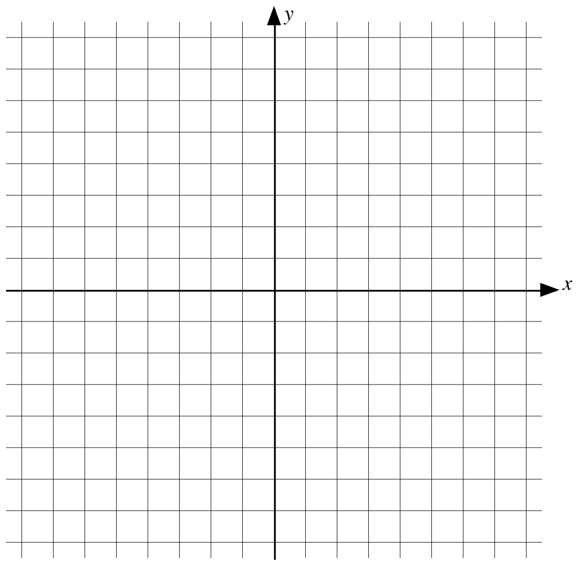
5. Each part of this problem describes a region of the *xy*-plane. Find the area of each region.

(a) The region lies within the curve $r = 1 + 2\cdot\cos(\theta)$ and outside the circle r = 2.



(b) The region lies inside both $r^2 = \cos(2\theta)$ and $r^2 = \sin(2\theta)$.



6. Find an equation for the plane that passes through the points $(1, 0, -1)$ and $(2, 1, 0)$ that is parallel to the line of intersection of the planes $x + y + z = 5$ and $3x - y = 4$.