SOLUTIONS.

Quiz #6

For each question, be careful to indicate your final answer **and where appropriate, show how you obtained it**.

1. In this problem you will always use the differential equation: $\frac{dQ}{dt} = t^2 - Q^2$.

(a) **(2 points)** Suppose that $Q(0) = -0.25$. Use Euler's method and $\Delta t = \frac{1}{2}$ to estimate $Q(2)$. Show your work and clearly indicate your final answer.

Current t	Current Q	dQ/dt	Rise	New Q
0	− 0.25	− 0.0625	−0.03125	− 0.28125
1/2	−0.28125	0.17089	0.08545	−0.1958
1	−0.1958	0.96166	0.48083	0.28503
1 1/2	0.28503	2.16875	1.08437	1.3694
2	1.3694			

FINAL ANSWER: $Q(2) =$ 1.3694

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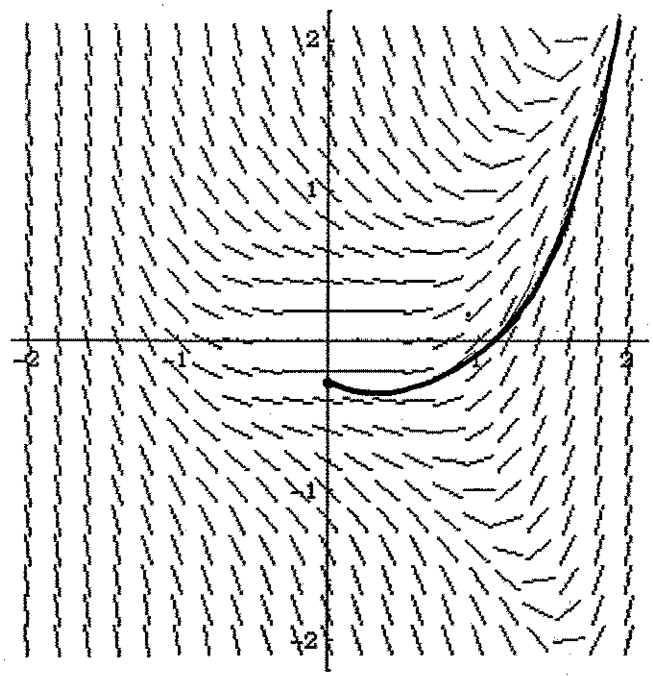
(b) **(2 points)** The slope field shown below corresponds to the differential equation $\frac{dQ}{dt} = t^2 - Q^2$. Suppose that $Q(0) = -0.25$. Carefully sketch the graph of $Q(t)$ over the interval $0 \le t \le 2$.



(c) **(2 points)** Is the estimate of $Q(2)$ produced in Part (a) an overestimate or an underestimate? Briefly (in a sentence or two) explain how you know.

It is an underestimate of the "true" value of Q(2).

This is because (as shown by the slope field) the function Q(t) is concave up over the interval concerned.

SOLUTIONS.

2. Use the technique of Separation of Variables to solve each of the following initial value problems. Clearly indicate your final answers and show your work. No work = no credit.

You should not use your calculator to find antiderivatives on Problem 2 of this quiz. You may use your calculator for arithmetic and to evaluate functions if you think this will be helpful.

(a) **(5 points)** $\dfrac{dw}{d\theta} = e^w \cdot \theta \cdot \cos(\theta^2)$ $w(\pi) = -1.$

$\boxed{u = -w \quad \frac{du}{dw} = -1}$

$$\int e^{-w}\, dw = \int \theta \cdot \cos(\theta^2) \cdot d\theta$$

$\boxed{u = \theta^2 \quad \frac{du}{d\theta} = 2\theta}$

$$-e^{-w} = \frac{1}{2}\sin(\theta^2) + C$$

To determine C, plug in π for θ and -1 for w.

$$-e = \frac{1}{2}\sin(\pi^2) + C$$

$$-e - \frac{1}{2}\sin(\pi^2) = C$$

$$C \approx -2.50313122$$

To solve for w:

$$e^{-w} = -\frac{1}{2}\sin(\theta^2) + 2.50313122$$

$$-w = \ln\left(-\frac{1}{2}\sin(\theta^2) + 2.50313122\right)$$

$$w = -\ln\left(-\frac{1}{2}\sin(\theta^2) + 2.50313122\right)$$

FINAL ANSWER: $w(\theta) = -\ln\left(-\frac{1}{2}\sin(\theta^2) + 2.50313122\right)$

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SOLUTIONS

Use the technique of Separation of Variables to solve each of the following initial value problems. Clearly indicate your final answers and show your work. No work = no credit.

You should not use your calculator to find antiderivatives on Problem 2 of this quiz. You may use your calculator for arithmetic and to evaluate functions if you think this will be helpful.

(b) (5 points) $y' \cdot x = (1-y)\cdot \ln(x)$ $y(1)=0.$

$\boxed{\begin{array}{l} u = 1-y \\ \frac{du}{dy} = -1 \end{array}}$ $\boxed{\begin{array}{l} u = \ln(x) \\ \frac{du}{dx} = \frac{1}{x} \end{array}}$

$$\int \frac{1}{1-y}\,dy = \int \frac{\ln(x)}{x}\,dx$$

$$-\ln(1-y) = \frac{1}{2}(\ln(x))^2 + C$$

To determine C, plug in 1 for x and 0 for y:

$$-\ln(1) = \frac{1}{2}(\ln(1))^2 + C$$

$$C = 0$$

Rearranging to make y the subject of the equation:

$$\ln(1-y) = \frac{-1}{2}(\ln(x))^2$$

$$1-y = e^{-\frac{1}{2}(\ln(x))^2}$$

$$y = 1 - e^{-\frac{1}{2}(\ln(x))^2}$$

FINAL ANSWER: $y(x) = \underline{1 - e^{-\frac{1}{2}(\ln(x))^2}}$