Additional Review Problems[1] for Quiz #6

1. Consider the differential equation $y' = x + y$. Use Euler's method with $\Delta x = 0.1$ to estimate $y(0.4)$ for the solution curves satisfying:

(a) $y(0) = 1$ **(b)** $y(-1) = 0$.

2. In this problem you will work with the differential equation $\frac{dy}{dx} = 2x$ with initial condition $y(0) = 1$.

(a) Use Euler's method with $\Delta x = 0.5$ to estimate $y(1)$.

(b) Use Euler's method with $\Delta x = 0.25$ to estimate $y(1)$.

(c) What is the formula for the exact value of $y(x)$?

(d) The error in Euler's method is directly proportional to Δx. Do your calculations in this problem support this?

3. In this problem, the differential equation you are interested in is:

$$\frac{dy}{dx} = \sin(x) \cdot \sin(y).$$

(a) Starting with $y(0) = 2$, and using $\Delta x = 0.1$, approximate $y(0.3)$.

(b) Starting instead with $y(0) = \pi$, and using $\Delta x = 0.1$, approximate $y(0.3)$.

4. Use Euler's method to find $B(2)$ starting with the differential equation $\frac{dB}{dt} = 0.05 \cdot B$, the initial value $B(1) = 1000$ and:

(a) $\Delta t = 1$ **(b)** $\Delta t = 0.5$ **(c)** $\Delta t = 0.25$.

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[1] The problems given here are adapted from *Calculus* by Gleason, Hughes-Hallet et al.

Answers

1.(a) $y(0.4) \approx 1.5282$.

1.(b) $y(0.4) \approx -1.4$

2.(a) $y(1) \approx 1.5$.

2.(b) $y(1) \approx 1.75$.

2.(c) $y(x) = x^2 + 1$ so $y(1) = 2$.

2.(d) The error in Part (a) is 0.5 and the error in Part (b) is 0.25. So, the error does certainly seem to be proportional to Δx. (In fact here, the error seems to be equal to Δx.)

For Problem #3, make sure your calculator is in RADIAN mode.

3.(a) $y(0.3) \approx 2.027$.

3.(b) $y(0.3) \approx \pi$.

4.(a) $B(2) \approx 1050$.

4.(b) $B(2) \approx 1050.63$.

4.(c) $B(2) \approx 1050.94$.