Additional Review Problems for Quiz #3

1. Evaluate the indefinite integral: $\int \frac{1}{x^2 - 8x + 17} dx$.

2. Evaluate the indefinite integral: $\int \frac{1}{8x^2 + 32x + 40} dx$.

3. Evaluate the indefinite integral: $\int \frac{1}{9x^2 - 18x + 10} dx$.

4. Evaluate the indefinite integral: $\int \frac{1}{4x^2 + 4x + 2} dx$.

5. Evaluate the indefinite integral: $\int \frac{2x^3 + x^2 - 25x + 13}{x + 4} dx$.

6. Evaluate the indefinite integral: $\int \frac{x^4 - 6x^3 - 7x^2 + 5x + 11}{x + 1} dx$.

7. Evaluate the indefinite integral: $\int \frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2 + 1} dx$.

8. Evaluate the indefinite integral: $\int \frac{x^3 - 5x^2 + 8x - 2}{x - 3} dx$.

Answers

1. The denominator of this rational function does not factor, so we will use the technique of completing the square to try to find an antiderivative. Completing the square gives that:

$$x^2 - 8x + 17 = (x-4)^2 + 1 = 1 + (x-4)^2.$$

So the integral can be rewritten and integrated using the u-substitution $u = x - 4$.

$$\int \frac{1}{x^2 - 8x + 17}\,dx = \int \frac{1}{1+(x-4)^2}\,dx = \int \frac{1}{1+u^2}\,du = \arctan(u) + C = \arctan(x-4) + C.$$

2. The denominator of this rational function does not factor, so we will use the technique of completing the square to try to find an antiderivative. Completing the square gives that:

$$8x^2 + 32x + 40 = 8 + 8(x+2)^2 = 8\left[1+(x+2)^2\right].$$

So the integral can be rewritten and integrated using the u-substitution $u = x + 2$.

$$\int \frac{1}{8x^2 + 32x + 40}\,dx = \frac{1}{8}\int \frac{1}{1+(x+2)^2}\,dx = \frac{1}{8}\int \frac{1}{1+u^2}\,du = \tfrac{1}{8}\arctan(u) + C = \tfrac{1}{8}\arctan(x+2) + C$$

3. The denominator of this rational function does not factor, so we will use the technique of completing the square to try to find an antiderivative. Completing the square gives that:

$$9x^2 - 18x + 10 = 9(x-1)^2 + 1 = 1 + 9(x-1)^2 = 1 + \left[3(x-1)\right]^2.$$

So the integral can be rewritten and integrated using the u-substitution $u = 3(x-1)$.

$$\int \frac{1}{9x^2 - 18x + 10}\,dx = \int \frac{1}{1+\left[3(x-1)\right]^2}\,dx = \int \frac{1}{1+u^2}\frac{du}{3} = \tfrac{1}{3}\arctan(u) + C = \tfrac{1}{3}\arctan(3(x-1)) + C$$

4. The denominator of this rational function does not factor, so we will use the technique of completing the square to try to find an antiderivative. Just completing the square doesn't give a very attractive result in this case, either (although you can still go ahead and use it). Slightly more inspired factoring gives that:

$$4x^2 + 4x + 2 = (2x+1)^2 + 1 = 1 + (2x+1)^2.$$

So the integral can be rewritten and integrated using the u-substitution $u = 2x + 1$.

$$\int \frac{1}{4x^2+4x+2}\,dx = \int \frac{1}{1+(2x+1)^2}\,dx = \int \frac{1}{1+u^2}\frac{du}{2} = \tfrac{1}{2}\arctan(u) + C = \tfrac{1}{2}\arctan(2x+1) + C$$

5. The integrand in this problem can be significantly simplified through polynomial division. Doing this with the quotient in this problem yields:

$$\frac{2x^3 + x^2 - 25x + 13}{x+4} = 2x^2 - 7x + 3 + \frac{1}{x+4}.$$

Integrating this gives the answer to the problem.

$$\int \frac{2x^3 + x^2 - 25x + 13}{x+4}\,dx = \int\left(2x^2 - 7x + 3 + \frac{1}{x+4}\right)dx = \frac{2}{3}x^3 - \frac{7}{2}x^2 + 3x + \ln(|x+4|) + C$$

6. The integrand in this problem can be significantly simplified through polynomial division. Doing this with the quotient in this problem yields:

$$\frac{x^4 - 6x^3 - 7x^2 + 5x + 11}{x+1} = x^3 - 7x^2 + 5 + \frac{6}{x+1}.$$

Integrating this gives the answer to the problem.

$$\int \frac{x^4 - 6x^3 - 7x^2 + 5x + 11}{x+1}\,dx = \int\left(x^3 - 7x^2 + 5 + \frac{6}{x+1}\right)dx = \frac{1}{4}x^4 - \frac{7}{3}x^3 + 5x + 6\ln(|x+1|) + C$$

7. The integrand in this problem can be significantly simplified through polynomial division. Doing this with the quotient in this problem yields:

$$\frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2+1} = x^2 - 3x + 2 + \frac{1}{1+x^2}.$$

Integrating this gives the answer to the problem.

$$\int \frac{x^4 - 3x^3 + 3x^2 - 3x + 3}{x^2+1}\,dx = \int\left(x^2 - 3x + 2 + \frac{1}{1+x^2}\right)dx = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + \arctan(x) + C$$

8. The integrand in this problem can be significantly simplified through polynomial division. Doing this with the quotient in this problem yields:

$$\frac{x^3 - 5x^2 + 8x - 2}{x - 3} = x^2 - 2x + 2 + \frac{4}{x - 3}.$$

Integrating this gives the answer to the problem.

$$\int \frac{x^3 - 5x^2 + 8x - 2}{x - 3}\,dx = \int \left(x^2 - 2x + 2 + \frac{4}{x - 3} \right) dx = \frac{1}{3}x^3 - x^2 + 2x + 4\ln\left(|x - 3|\right) + C$$