Math 122, Fall 2008. Answers to Unit Test 1 Review Problems – Set A.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $\int \frac{x \cdot e^{x^2}}{2+e^{x^2}} \cdot dx = \frac{1}{2} \cdot \ln\left(2+e^{x^2}\right) + C$ (Try $u = 2 + e^{x^2}$.)

1.(b) $\int \frac{24 \cdot x^3 + 4}{\left(3 \cdot x^4 + 2 \cdot x + 1\right)^9} \cdot dx = \frac{-1}{4} \cdot \left(3 \cdot x^4 + 2 \cdot x + 1\right)^{-8} + C$ (Try $u = 3x^4 + 2x + 1$.)

1.(c) $\int \frac{1}{2 \cdot \sqrt{x}} \cdot \frac{1}{(1+x)} \cdot dx = \tan^{-1}\left(\sqrt{x}\right) + C$ (Try $u = \sqrt{x}$.)

1.(d) $\int \frac{1}{x \cdot \ln(x)} \cdot dx = \ln(\ln(x)) + C$ (Try $u = \ln(x)$.)

1.(e) $\int \frac{1}{9+x^2} \cdot dx = \frac{1}{3} \cdot \tan^{-1}\left(\frac{x}{3}\right) + C$ (Factor out the 9 and then try $u = \frac{x}{3}$.)

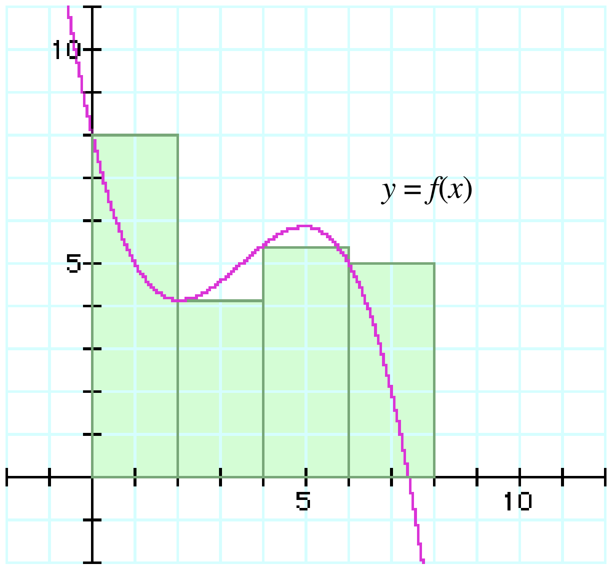
1.(f) $\int \frac{-2x}{\sqrt{1-x^2}} \cdot dx = 2 \cdot \sqrt{1-x^2} + C$ (Try $u = 1 - x^2$.)

2.(a) The sum $\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$ represents the approximation to the value of the "true value" of the integral:

$$\int_0^8 f(x) \cdot dx$$

that you would get if you used 4 rectangles (each with a width $\Delta x = 2$) to approximate the area under the curve $y = f(x)$ between $x = 0$ and $x = 8$.

2.(b) The area that would have the same numerical value as the sum $\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$ is shaded in the diagram shown below.



2.(c) The function values needed to approximate the sum $\sum_{k=0}^{3} f(k \cdot 2) \cdot 2$ are given in the table below.

K	$2 \cdot k$	$f(2 \cdot k)$
0	0	8
1	2	4.142857
2	4	5.428571
3	6	5

The numerical value of the sum is:

$$\sum_{k=0}^{3} f(k \cdot 2) \cdot 2 = (8 + 4.142857 + 5.42857 + 5) \cdot 2 = 45.142857.$$

2.(d) Using 100 rectangles to approximate the area between the curve $y = f(x)$ and the x-axis from $x = 0$ to $x = 8$, the relevant commands for a TI-83 calculator are:

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(8-0)/100 [STO⇨] W

Y1 = (-1/7)*(X-1)*(X-3.5)*(X-6) + 5

sum(seq(Y1(0+K*W)*W,K,0,99))
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The numerical value of this approximation of the area under the curve is: 34.76388571.

2.(e) The symbolic expression that will represent the precise value of the area beneath the graph $y = f(x)$ between $x = 0$ and $x = 8$ is:

$$\textit{Precise Value of Area} = \int_{0}^{8} f(x) \cdot dx.$$

2.(f) The equation for the function $f(x)$ is:

$$f(x) = \tfrac{1}{7}\cdot(x-1)\cdot(x-3.5)\cdot(x-6)+5 = \tfrac{-1}{7}\cdot x^3 + \tfrac{3}{2}\cdot x^2 - \tfrac{61}{14}\cdot x + 8.$$

An anti-derivative, $F(x)$, of $f(x)$ is given by:

$$F(x) = \tfrac{-1}{28}\cdot x^4 + \tfrac{1}{2}\cdot x^3 - \tfrac{61}{28}\cdot x^2 + 8\cdot x + C.$$

Using this anti-derivative the precise value of the area will be given by:

$$\textit{Precise Value of Area} = \int_0^8 f(x)\cdot dx = F(8) - F(0) \approx .34.28571429$$

3.(a) $\int e^{-w}\cdot\cos(2w)\cdot dx = \tfrac{2}{5}e^{-w}\sin(2w) - \tfrac{1}{5}e^{-w}\cos(2w) + C.$

3.(b) $\int_1^{\sqrt{3}} \arctan\left(\tfrac{1}{\theta}\right)\cdot d\theta = \tfrac{\pi\sqrt{3}}{6} - \tfrac{\pi}{4} + \tfrac{1}{2}\ln(2).$ (Hint: Let $v' = 1$.)

3.(c) $\int \sqrt{x}\cdot\ln(x)\cdot dx = \tfrac{2}{3}x^{\frac{3}{2}}\cdot\ln(x) - \tfrac{4}{9}x^{\frac{3}{2}} + C.$

3.(d) $\int_{\sqrt{\frac{\pi}{2}}}^{\sqrt{\pi}} t^3\cdot\cos(t^2)\cdot dt = \tfrac{-1}{2} - \tfrac{\pi}{4}.$ (Hint: First make the substitution $w = t^2$.)

3.(e) $\int (\ln(q))^2\cdot dq = q\cdot(\ln(q))^2 - 2q\cdot\ln(q) + 2q + C.$

3.(f) $\int e^{\sqrt{x}}\cdot dx = 2\sqrt{x}\cdot e^{\sqrt{x}} - 2e^{\sqrt{x}} + C.$ (Hint: First make the substitution $w = \sqrt{x}$.)

4.(a) $\int_0^2 (f(x))^2\cdot f'(x)dx = \left[\tfrac{1}{3}(f(x))^3\right]_0^2 = \tfrac{1}{3}(1000 - \pi^3).$

4.(b) $\int_2^4 x\cdot f'(x)dx = [x\cdot f(x)]_2^4 - \int_2^4 f(x)dx = 4\sqrt{2} - 28.$

4.(c) $\int_0^2 2x\cdot f'(x^2)dx = \left[f(x^2)\right]_0^2 = \sqrt{2} - \pi.$

4.(d) $\int_0^2 x^2\cdot f'(x^2)dx = \left[x\cdot\tfrac{1}{2}\cdot f(x^2)\right]_0^2 - \tfrac{1}{2}\int_0^2 f(x^2)dx = \sqrt{2} - \tfrac{13}{2}.$

5.(a) $\int \cos^4(x)\cdot\tan^3(x)\cdot dx = \tfrac{1}{4}\sin^4(x) + C.$

5.**(b)** $\int \sec^6(\theta)\cdot d\theta = \tfrac{1}{5}\tan^5(\theta) + \tfrac{2}{3}\tan^3(\theta) + \tan(\theta) + C.$

5.(c) $\int \tan^2(w) \cdot dw = \tan(w) - w + C.$

5.(d) $\int \sin^6(u) \cdot \cos^3(u) \cdot du = \tfrac{1}{7}\sin^7(u) - \tfrac{1}{9}\sin^9(u) + C.$

5.(e) $\int p \cdot \cos^2(p) \cdot dp = \tfrac{1}{4}p^2 + \tfrac{1}{4}p \cdot \sin(2p) + \tfrac{1}{8}\cos(2p) + C.$

5.(f) $\int \left(1 - \sin(\theta)\right)^2 \cdot d\theta = \tfrac{3}{2}\theta + 2\cos(\theta) - \tfrac{1}{4}\sin(2\theta) + C.$

6.(a) $\int \frac{r^2}{r+1} dr = \tfrac{1}{2}r^2 - r + \ln(|r+1|) + C.$

6.(b)

$$\int \frac{1}{x^2 - x + 1} dx = \int \frac{1}{\frac{3}{4} + \left(x - \frac{1}{2}\right)^2} dx = \frac{4}{3}\int \frac{1}{1 + \left(\frac{2}{\sqrt{3}}\left(x - \frac{1}{2}\right)\right)^2} dx = \tfrac{2}{\sqrt{3}}\arctan\left(\tfrac{2}{\sqrt{3}}\left(x - \tfrac{1}{2}\right)\right) + C.$$

6.(c) $\int \frac{10}{(x-1)(x^2+9)} dx = \ln(|x-1|) - \tfrac{1}{2}\ln(x^2+9) - \tfrac{1}{3}\arctan\left(\tfrac{x}{3}\right) + C.$

6.(d) $\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx = 2 \cdot \ln(|x|) + \frac{1}{x} + 3 \cdot \ln(|x+2|) + C.$

6.(e) $\int \frac{x^3 + x}{x - 1} dx = \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x + 2 \cdot \ln(|x-1|) + C.$

6.(f) $\int \frac{2x^3 + 5x}{x^4 + 5x^2 + 4} dx = \tfrac{1}{2}\ln\left(\left|x^4 + 5x^2 + 4\right|\right) + C.$ (Use u-substitution.)

7.(a) $\int_0^2 y^3 \cdot \sqrt{y^2 + 4} \cdot dy = \tfrac{64}{15}\left(\sqrt{2} + 1\right).$

7.(b) $\int \sqrt{1 - 4x^2} dx = \tfrac{1}{4}\sin^{-1}(2x) + 2x \cdot \sqrt{1 - 4x^2} + C.$

7.(c) $\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2 - 1}} dx = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}.$

7.(d) $\int w \cdot \sqrt{1 - w^2} dw = \tfrac{-1}{3}\left(1 - w^2\right)^{\frac{3}{2}} + C.$

7.(e) $\int w \cdot \sqrt{1 - w^4} dw = \tfrac{1}{4}\sin^{-1}\left(w^2\right) + \tfrac{1}{4}w^2 \cdot \sqrt{1 - w^4} + C.$

7.(f) $\int \frac{p^5}{\sqrt{p^2+2}}\,dp = \frac{1}{15}\left(3p^4 - 8p^2 + 32\right)\cdot\sqrt{p^2+2} + C.$

8.(a) Left hand sum = 6.976174136.

8.(b) Right hand sum = 6.994482433.

8.(c) Midpoint sum = 6.988646849.

8.(d) Trapezoid sum = 6.9885902.

8.(e) Simpson's rule = 6.988620174. Note that this is equal to one third of the answer to 8(d) plus two-thirds of the midpoint sum that uses 400 rectangles.

9.(a) Converges to 1.

9.(b) Converges to 2.

9.(c) Diverges.

9.(d) This is not an improper integral. It converges to $1 - e$.

9.(e) Converges to $\frac{\pi}{4}$.

9.(f) Diverges.

10.(a) No it is not possible because the graph changes concavity part way through the interval.

10.(b) Trapezoid rule gives 0.881839.

10.(c) Midpoint rule gives 0.882202.

10.(d) Using $K = 2$, $n = 366$ rectangles would be sufficient. If you used a different value of K you will get a different value of n.

10.(e) Using $K = 2$, $n = 259$ rectangles would be sufficient. If you used a different value of K you will get a different value of n.

10.(f) Simpson's rule gives 0.882081.

10.(g) Using $K = 12$, $E_S \leq 0.0000134$.