Calculating Volumes

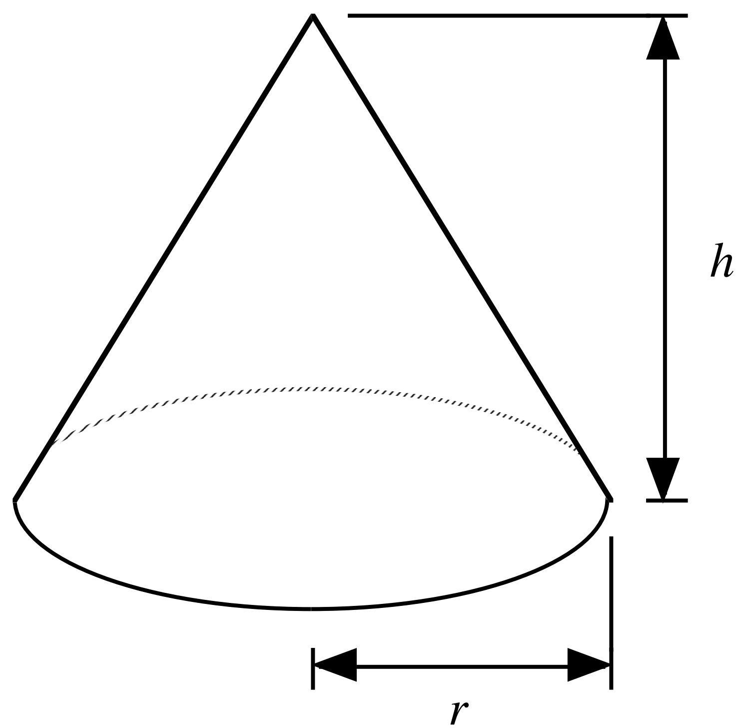
Setting Up and Evaluating
Integrals to Calculate Volumes

Volume of a Cone

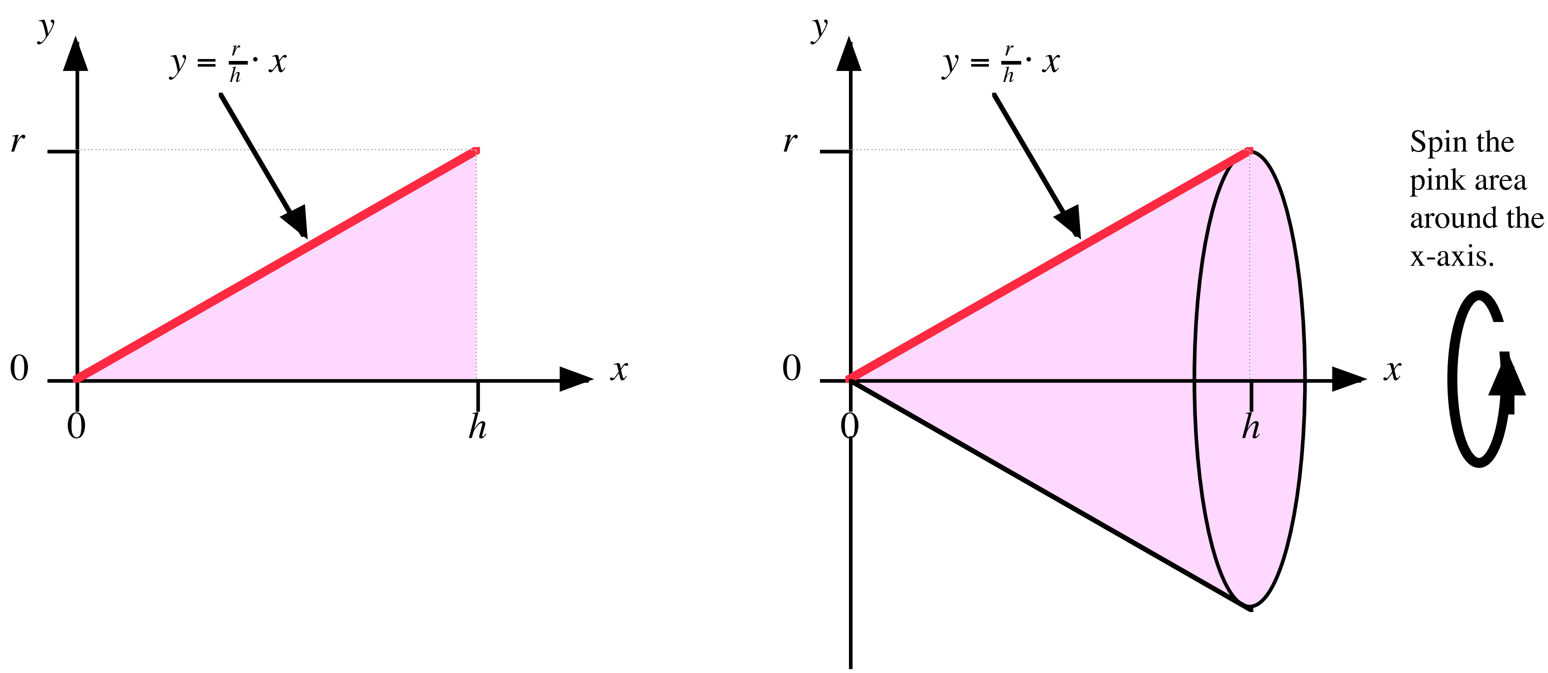
- The formula for the volume of a cone is:

$$V = \frac{1}{3}\pi \cdot r^2 \cdot h$$

- How did anyone come up with this formula?



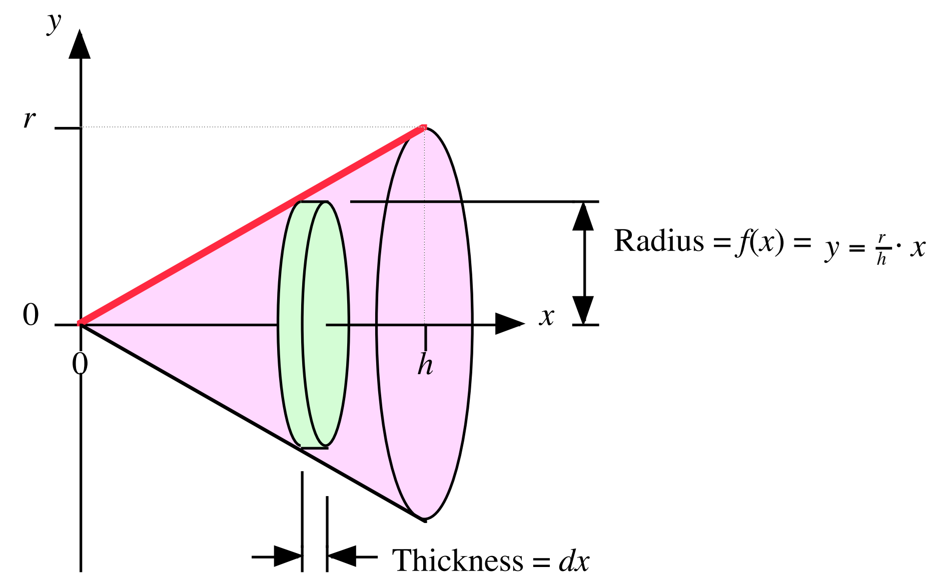
Visualizing the Volume



Setting Up the Integral

- **Step 1:** Break up the conical volume.

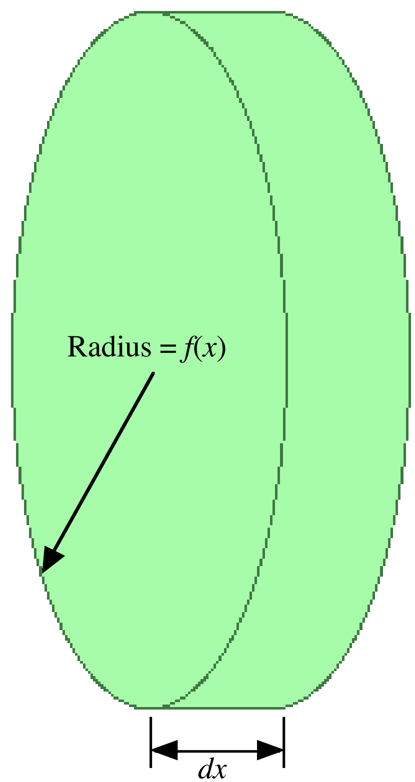
The idea will be to approximate the conical volume with a collection of regular shapes (in much the same way as you broke up the plume of radioactive material in the Chernobyl handout).



Setting Up the Integral

- **Step 2:** Determine a formula for the volume of each of these regular shapes.

$$\pi \cdot \left[f(x) \right]^2 \cdot dx = \frac{\pi \cdot r^2}{h^2} \cdot x^2 \cdot dx$$



Setting Up the Integral

- **Step 3:** Create an integral that combines all of the volumes of the regular shapes together to give the total volume.
- **Step 4:** Work out the integral. The result will be the total volume of the shape.

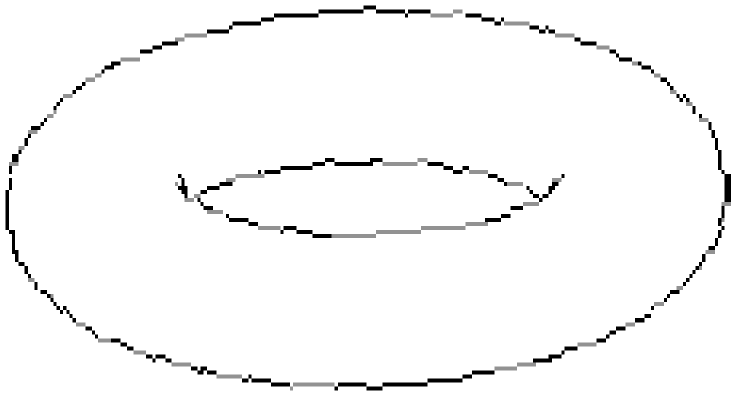
$$\int_0^h \frac{\pi \cdot r^2}{h^2} \cdot x^2 \cdot dx$$

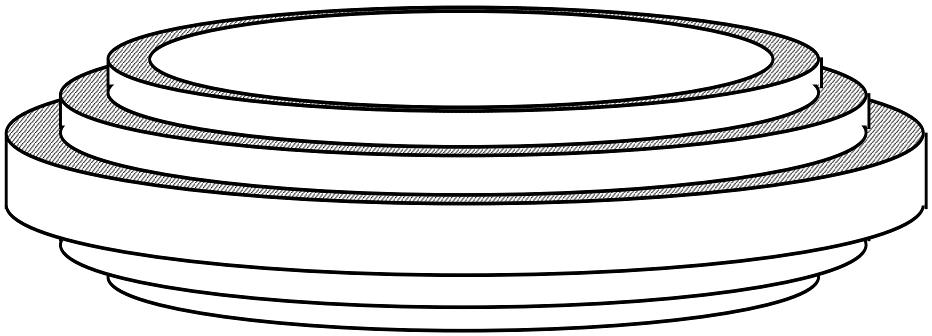
$$= \left[\frac{\pi \cdot r^2}{3 \cdot h^2} \cdot x^3 \right]_0^h$$

$$= \frac{\pi \cdot r^2}{3 \cdot h^2} \cdot h^3$$

$$= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$

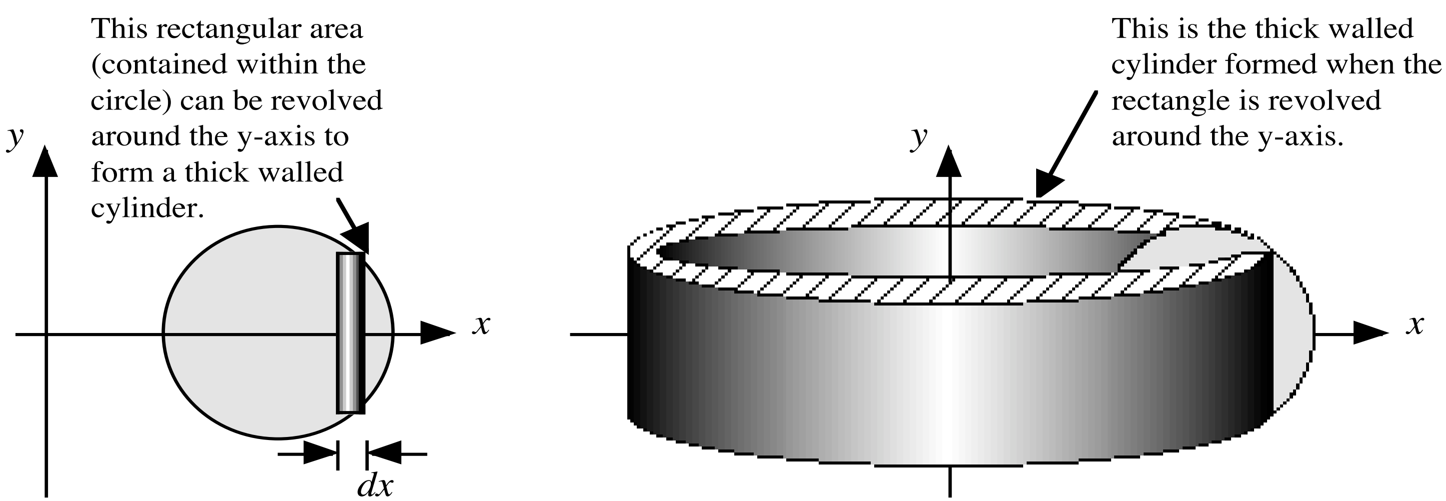
Volume of a Donut





- A solid donut (a torus) can be approximated by a collection of “nested, hollow cylinders”

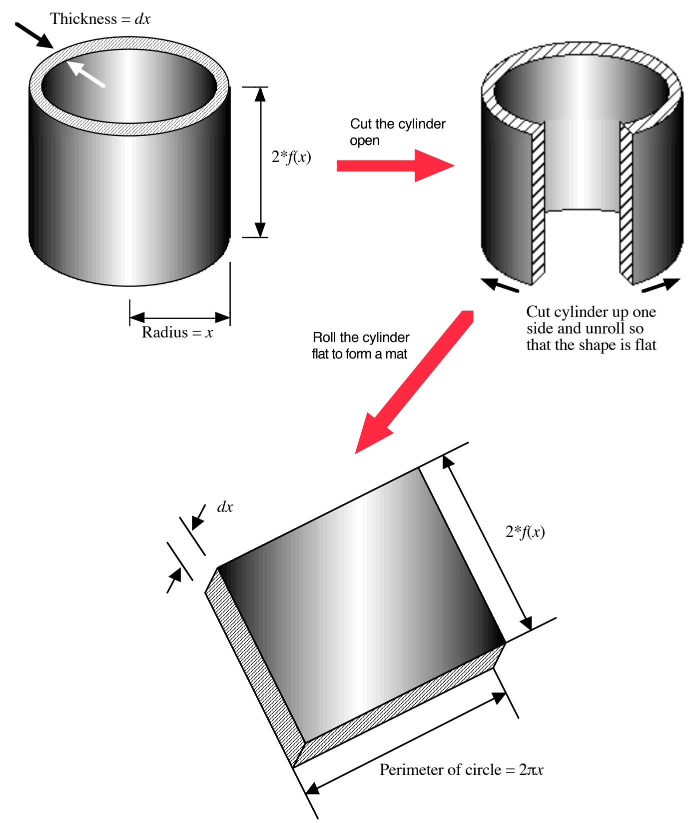
Volume of a Donut



- The “nested, hollow cylinders” are created by taking rectangles that cover the circle and rotating each rectangle around the *y*-axis

Formula for a Nested Cylinder

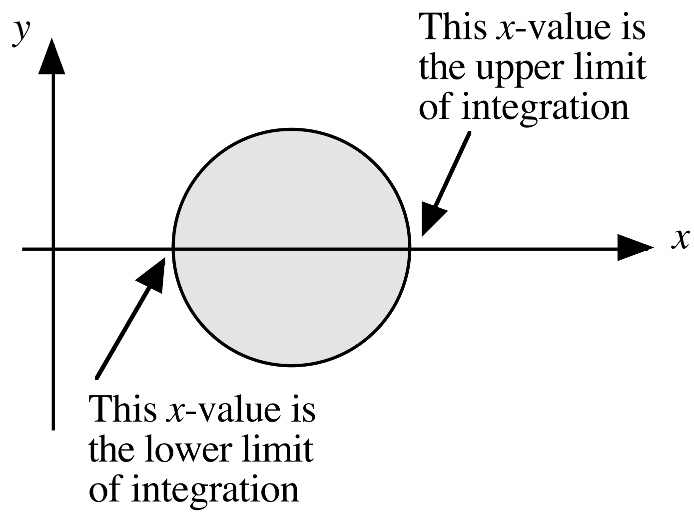
- Cut the cylinder open
- Unroll the cylinder so that the shape is flat
- The volume of the shape is length times width times thickness (dx)



Set Up the Integral

- To determine the limits of integration, you have to look at the actual area that is revolved around either the *x*- or the *y*-axis to create the solid.

- The **lower limit of integration** is the *x*-value where the area begins. The **upper limit of integration** is the *x*-value where the area ends.



The Integral for the Volume

$$\int_{R-r}^{R+r} (2\cdot\pi\cdot x)\cdot 2\cdot\sqrt{r^2-(x-R)^2}\cdot dx$$