Math 122 **Fall 2008**

Handout 5(a): Design of the USS Scorpion

Since the end of World War 2, only two U.S. Navy vessels have been lost with all hands (meaning that all of the crew members perished along with the ship.) These vessels were the nuclear submarines USS Thresher (SSN-593) and USS Scorpion (SSN-589). (See Figure 1[1].)

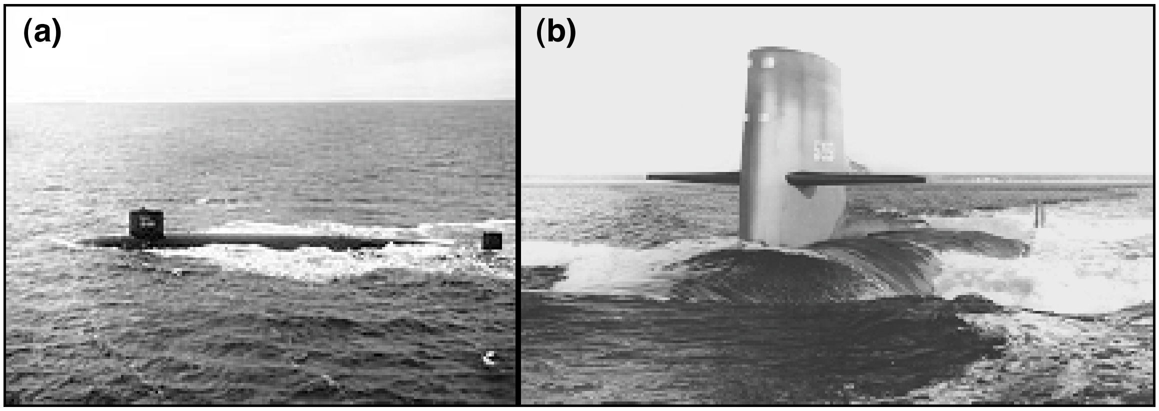

Figure 1: (a) USS Thresher (SSN-593). (b) USS Scorpion (SSN-589).

The USS Scorpion is believed to have sunk on May 22, 1968. The cause of the sinking is clear and mysterious at the same time. The cause of the sinking was clearly a serious breach of the hull of the submarine. Seawater flooded the sub and unable to maintain buoyancy it sank to the bottom of the ocean. (The remains of the USS Scorpion were located in 1968 about 400 miles east of the Azores in 10,000 feet of water.) However, exactly what caused the hull breach is one of the great maritime mysteries of the 20th century.

(a) The USS Scorpion was launched on December 29, 1959 (see Figure 2). As you can see from Figure 2 the hull of the USS Scorpion appeared to be almost perfectly circular when examined from the front or rear of the submarine. Based on this, suggest a method that naval architects might have used to calculate the volume of the USS Scorpion.

[1] Image sources: (a) http://www.history.navy.mil/ (b) http://www.txoilgas.com/589.htm

(b) Figure 11[2] shows a cross-sectional view of a *Skipjack* class submarine (the class of submarines that the USS Scorpion belonged to). Use Figure 11 to set up a pair of integrals whose numerical value will equal the volume of the submarine. (All of the dimensions shown in Figure 11 are in meters, so these integrals will give the volume of the submarine in units of cubic meters.)

(c) Evaluate the integrals that you set up in Part (b) to find the volume of the USS Scorpion in cubic meters.

[2] The basic image for this diagram was obtained from: http://www.mediacen.navy.mil/
The dimensions used for the diagram are those of the USS Scorpion and were obtained from: http://www.history.navy.mil/danfs/s/scorpion.htm

(d) About 65% of the volume inside the hull of the USS Scorpion was filled with air with the remaining 35% of the volume occupied by equipment, fixtures, etc. The mass of the hull and all of the equipment, fixtures, etc inside the hull was about 2790 metric tons[3]. Calculate the total mass of the USS Scorpion in metric tons. (Note: 1 cubic meter of air has a mass of 1294.64 grams.)

(e) One cubic meter of seawater has a mass[4] of approximately 1027.8 kg (1.0278 metric tons). Calculate the mass of the seawater that has the same volume as the USS Scorpion and use Archimedes principle[5] to calculate the buoyant force (in units of metric tons) experienced by the USS Scorpion.

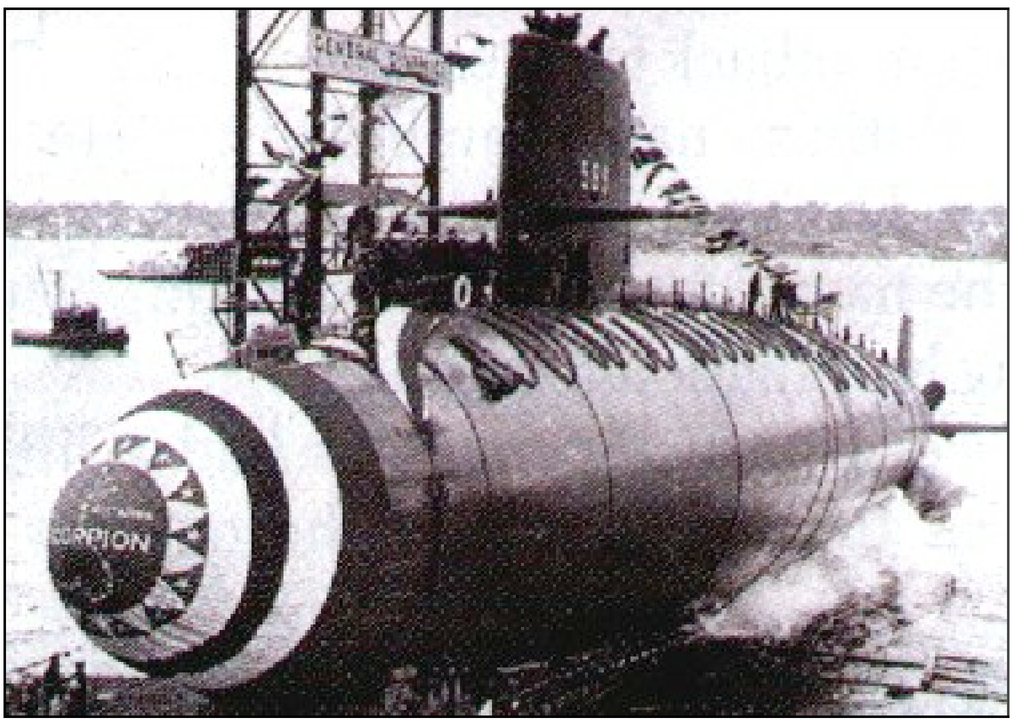

Figure 2: The launch of the USS Scorpion (SSN-589) at the Electric Boat Division, General Dynamics Corporation located in Portsmouth, NH. The USS Scorpion was launched on December 29, 1959.

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[3] Source: http://www.history.navy.mil/danfs/s/scorpion.htm

[4] Source: http://www.seaworld.org/Physics/bouyancy.htm

[5] One version of Archimedes principle states: "The net buoyant force on an object is equal to the weight of fluid displaced by the object minus the weight of the object itself."

(f) Submarines submerge by expelling air from large internal ballast tanks, and allowing these tanks to fill with water (see Figure 12[6]). Calculate how much water the USS Scorpion would have to allow into its ballast tanks in order to gain "neutral buoyancy" – that is, a buoyant force equal to zero. Remember that to let water into the ballast tanks, air must be expelled. You should take this into account in your calculations.

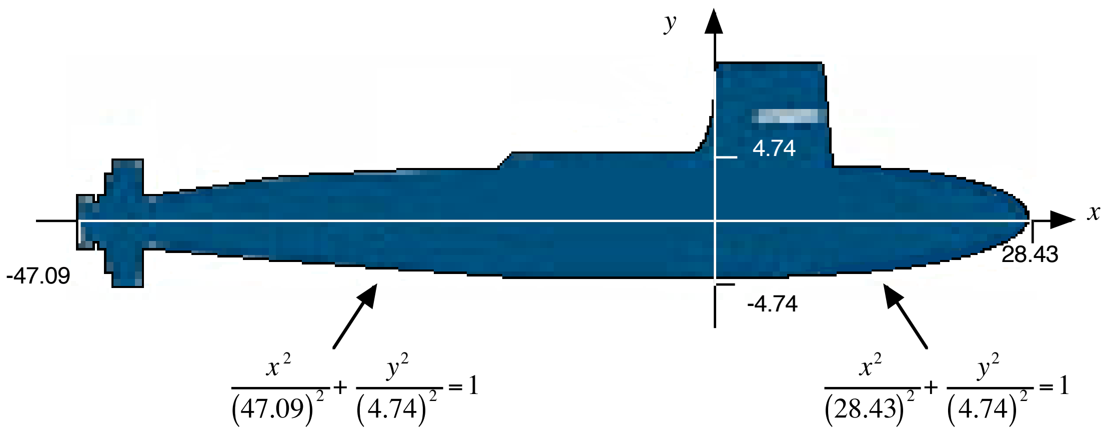

Figure 12: Mathematical model of the hull of the USS Scorpion (SSN-589). Note that the conning tower (or "sail" in US Navy parlance) has not been included in the volume as the conning tower is not water-tight, and the space inside the conning tower is not used to accomodate any of the equipment or crew of the submarine. All dimensions are in meters.

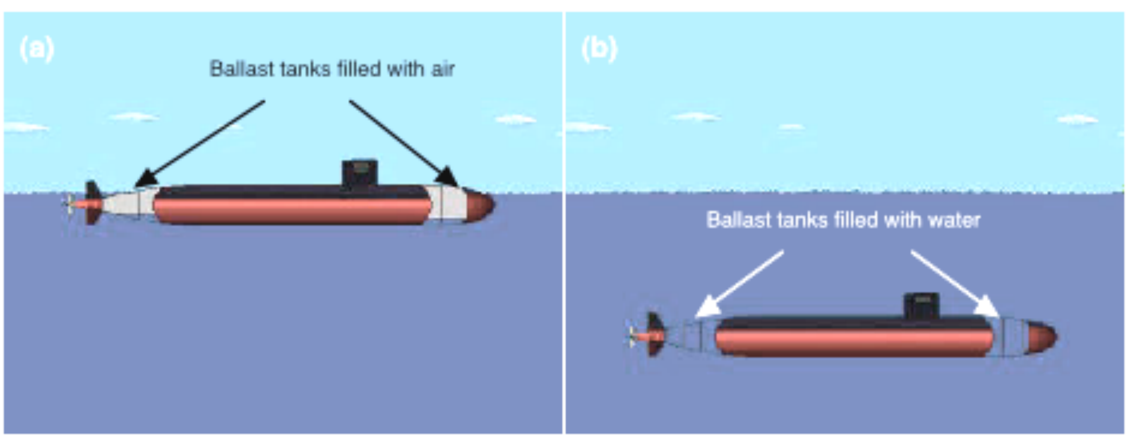

Figure 13: How a submarine submerges and surfaces. (a) When the ballast tanks are filled with air, the bouyant force keeps the submarine floating at the surface. (b) When the captain of the submarine wishes to submerge, air is removed from the ballast tanks, which are allowed to fill with sea water. As the mass inside the hull of the submarine increases, the bouyant force is no longer sufficient to keep the submarine at the surface and it sinks beneath the water.

[6] Image source: http://www.omsi.edu/explore/submarine/work.cfm