

Recitation Handout 19: Describing Curves Using Parametric Equations

This handout was designed to help you to develop an understanding of how parametric equations can be used to describe a curve in the x - y plane.

You will complete this activity in teams of two, three or four. In order to complete the activity, you will need the following pieces of equipment:

1. An "Etch-A-Sketch" drawing toy.
2. A stopwatch.

Part 1: The Parametric Equations that Describe a Circle

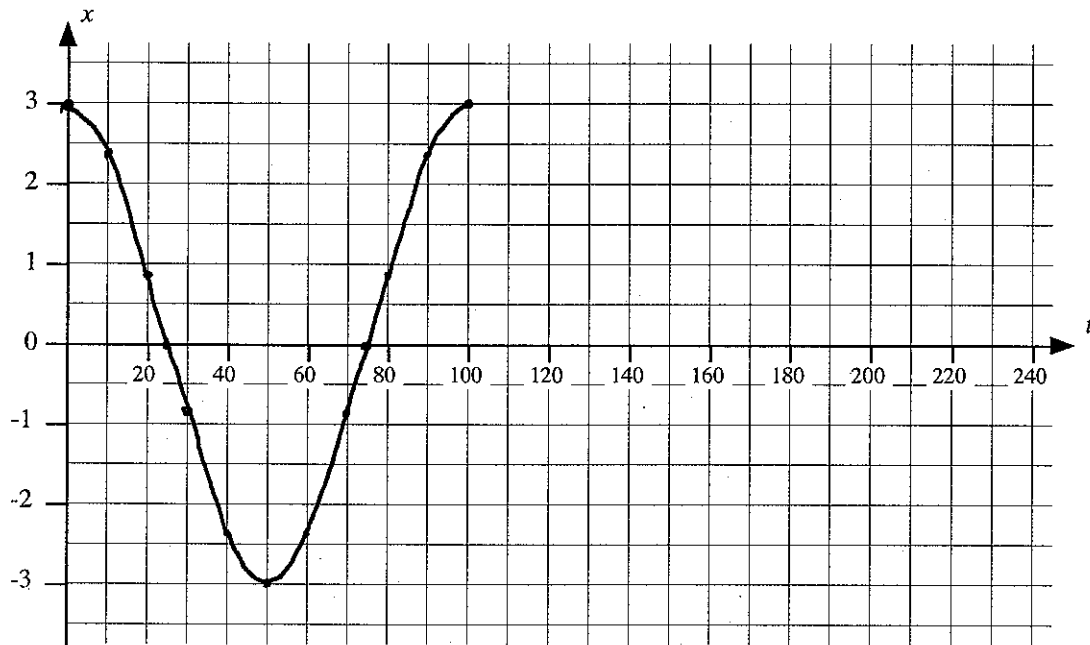
Here is what you will do as you complete the first part of this activity:

1. One person in the team will be the "drawer." It is their job to trace out a circle of **radius 3** on the Etch-A-Sketch working in 5-second increments. Start from the point $(3, 0)$ and work counterclockwise.
2. One person in the team will be the "timer." It is their job to operate the stopwatch and tell the drawer when each 5-second increment starts and ends.
3. Any remaining people in the team are "surveyors." It is their job to estimate the x - and y -coordinates of the point at the end of the circle at the conclusion of each 5-second increment and record this information below.

Time	x	y	Time	x	y
0	3	0	55	-2.85	-0.93
5	2.85	0.93	60	-2.43	-1.76
10	2.43	1.76	65	-1.76	-2.42
15	1.76	2.42	70	-0.93	-2.85
20	0.93	2.85	75	0	-3
25	0	3	80	0.93	-2.85
30	-0.93	2.85	85	1.76	-2.42
35	-1.76	2.42	90	2.42	-1.76
40	-2.43	1.76	95	2.85	-0.93
45	-2.85	0.93	100	3	0
50	-3	0	105		

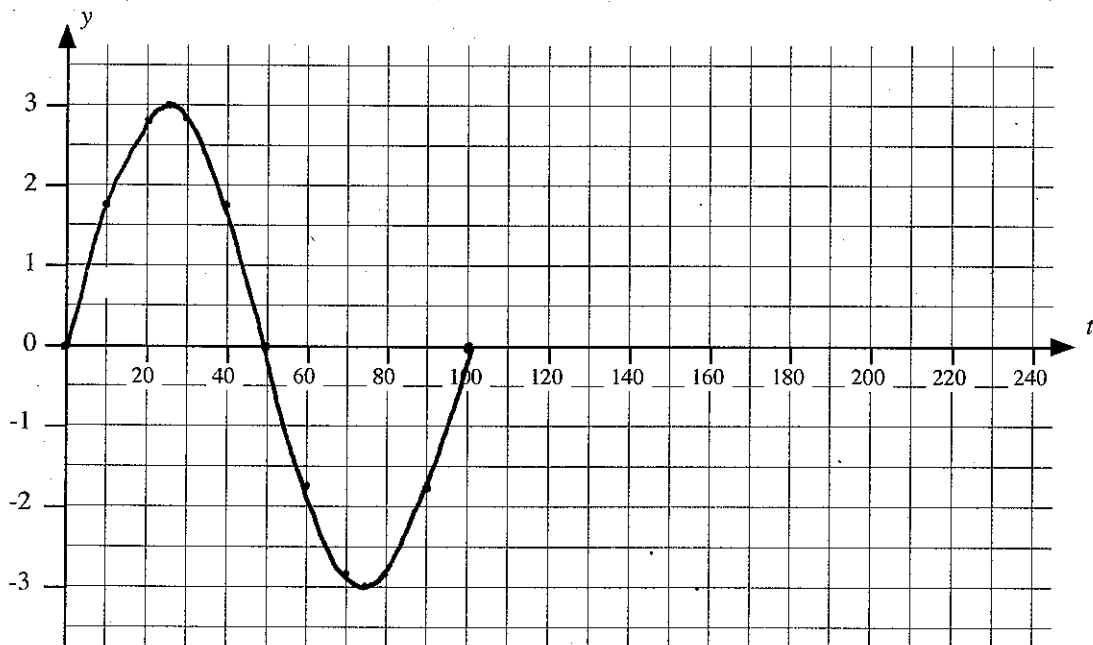
SOLUTIONS

Axes for plotting a graph showing x versus t .



Formula: $x(t) = 3 \cdot \cos\left(\frac{2\pi}{100} \cdot t\right)$

Axes for plotting a graph showing y versus t .

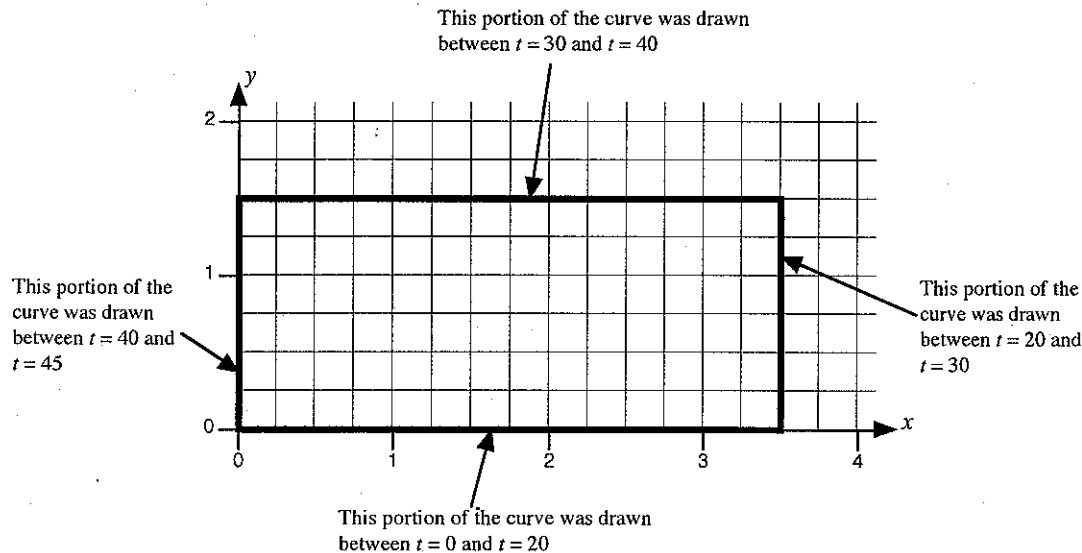


Formula: $y(t) = 3 \cdot \sin\left(\frac{2\pi}{100} \cdot t\right)$

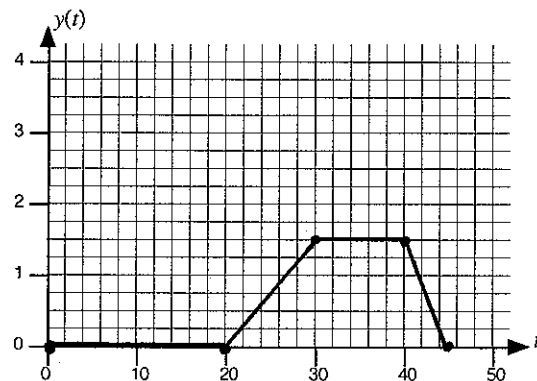
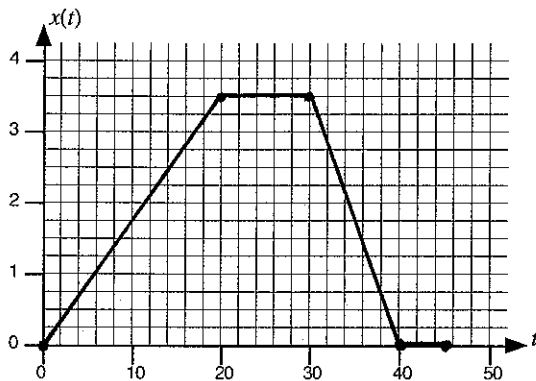
SOLUTIONS

Part 2: The Parametric Equations that Describe a Rectangle

Imagine that the following rectangular curve was traced out in the plane using the Etch-A-Sketch. The Etch-A-Sketch artist started at the point $(0, 0)$ and worked in a counter-clockwise direction. The amount of time that the artist needed to draw each part of the rectangular curve is shown on the diagram below.



In the same way that you created graphs showing the x -coordinate and the y -coordinates as functions of time for the circle, use the axes provided below to sketch the graphs of $x(t)$ and $y(t)$ that the Etch-A-Sketch artist could have created from the rectangular curve.



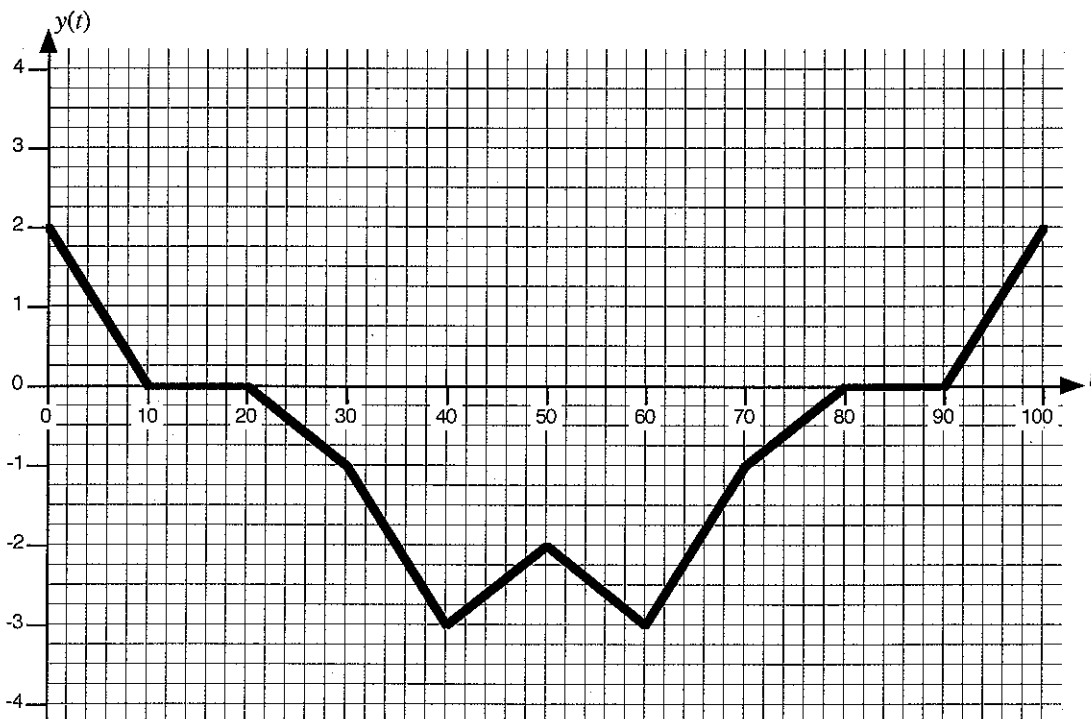
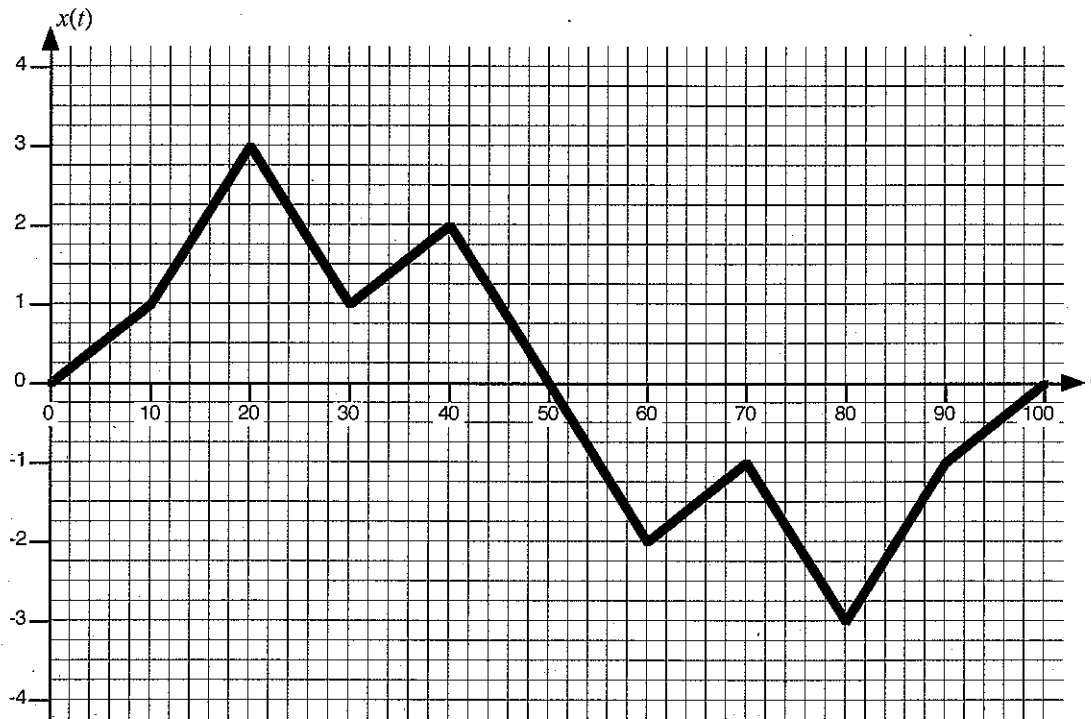
In the space provided below, write down a formula for $x(t)$. (Hint: use a function defined in pieces.)

$$x(t) = \begin{cases} \frac{3.5}{20} t & , \quad 0 \leq t < 20 \\ 3.5 & , \quad 20 \leq t < 30 \\ -\frac{3.5}{10} t + 14 & , \quad 30 \leq t < 40 \\ 0 & , \quad 40 \leq t \leq 45 \end{cases}$$

SOLUTIONS

Part 3: Reversing the Process – Drawing a Curve given $x(t)$ and $y(t)$

The two graphs given below show the graphs of the $x(t)$ and $y(t)$ functions for a curve in the x - y plane. Use these graphs and the axes provided below to sketch the curve that these graphs define.



SOLUTIONS.

