

**Handout 12: Review Problems for Unit Test 2**

The topics that will be covered on Unit Test 2 are as follows.

- Areas between curves.
- Calculating volumes (non-rotation) using integrals.
- Volumes of revolution – disk method.
- Volumes of revolution – washer method.
- Volumes of revolution – shell method.
- Arc length.
- Using integrals to calculate masses.
- Center of mass.
- Using integrals to calculate work in physics.
- Using integrals to calculate hydrostatic force.
- Euler's method.
- Slope fields.
- Equilibrium solutions.
- Separation of variables.
- Integrating factors.
- Second order, homogeneous differential equations with constant coefficients.
- Method of Undetermined Coefficients.

This (roughly) covers Chapter 7 of the textbook plus the additional topics on differential equations that have been covered in lecture and recitation.

1. Calculate the volume of the solid whose base is the region bounded by:

- $y = e^x$ ,
- the  $x$ -axis, and,
- the lines  $x = 0$  and  $x = 1$

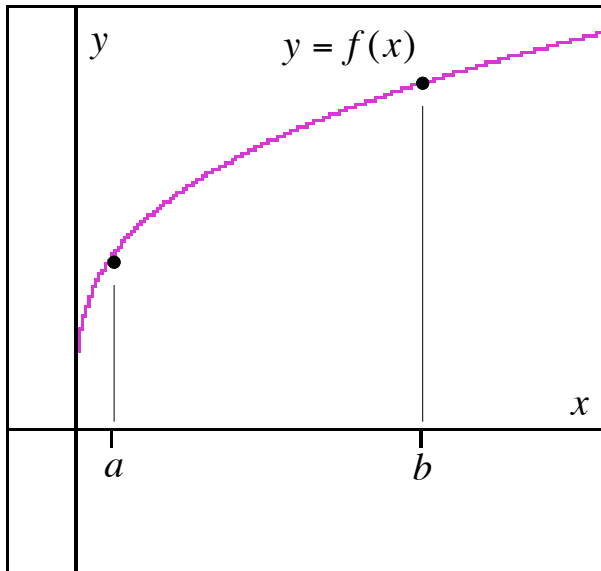
and whose cross-sections are equilateral triangles that are perpendicular to the  $x$ -axis.

2. Consider the function  $y = \frac{\sqrt{x^3}}{1+x^4}$  for  $0 \leq x < \infty$ . Let  $R$  be the region bounded by:

- The line  $x = 0$ , the line  $y = 0$ , and the curve  $y = \frac{\sqrt{x^3}}{1+x^4}$ .

Find the **exact** volume of the solid that is generated when the region  $R$  is revolved around the  $x$ -axis.

3. The following graph shows the function  $f(x)$  and the points  $x = a$  and  $x = b$  ( $a < b$ ). Three quantities are defined using this graph as follows:



**Quantity I:**  $QI = b - a$

**Quantity II:**  $QII = \int_a^b \sqrt{1 + [f'(x)]^2} dx$

**Quantity III:**  $QIII = \sqrt{(b - a)^2 + (f(b) - f(a))^2}$

Which ONE of the following MUST be true?

(i)  $QI < QII < QIII$

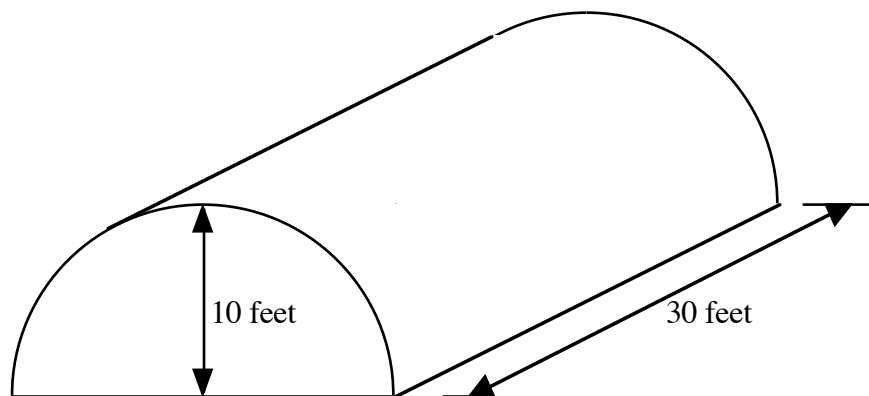
(ii)  $QIII < QI < QII$

(iii)  $QII < QI < QIII$

(iv)  $QI < QIII < QII$

(v)  $QII < QIII < QI$

4. The storage shed shown in the diagram below is in the shape of a half cylinder. The radius of the cylinder is 10 feet and the length of the shed is 30 feet. Suppose the shed is completely filled with sawdust. The density of the sawdust at a height of  $h$  feet above the floor is equal to  $(20 - h)$  pounds per cubic foot. If sawdust is extracted through a vent in the top of the roof, calculate the amount of work that must be done to clear the shed of sawdust when it is completely full.



5. Find solutions to the differential equations below, subject to the given initial conditions. In each case, *demonstrate that your answer is correct* by providing step-by-step work to show how your answer was obtained. If you do not provide any work to justify your answer, you will get zero credit, even if you circle the correct response.

(a)  $\frac{dP}{dt} = 4P - 8$   $P(0) = 20.$

(b)  $\frac{dy}{dx} + x^2 \cdot y = e^{\frac{-1}{3}x^3}$   $y(0) = 1.$

6. Find the formula for  $y(x)$ , the solution of initial value problem given below. Your final answer should contain no unspecified constants.

$$y'' - 5y' + 6y = e^x \qquad y(0) = 0 \qquad y'(0) = 1.$$