21112 (Calculus 2) Lecture 9 - Trigonometric functions - Episode 1: A review

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With some practice integrating standard polynomials, $\ln(x)$ and e^x -type functions, we look ahead to extending the class of functions we can operate on. We move on to trigonometric functions, with an eye to reviewing basic concepts.

MANY diagrams in this lecture.

The well-known formula for the circumference of a circle of radius r is

$$C = 2\pi r \tag{1}$$

If walked around in a circle, then you have swept out a total angle of 360 degrees. If you sweep around 180 degrees, then correspondingly you have traveled a distance of half of the circumference. This argument can be extended to any angle you can think of. So, we would like to have a formula like

$$Circumference(\theta) = \theta r \tag{2}$$

However, we need to have this formula match up with our previous one, especially when we have gone around 360 degrees ('We're going to turn this team around 360 degrees...'). We also need a dimensionless 'unit' to measure angle traveled.We will call this a *radian*:

$$(2\pi\ radians) * r = Circumference(360\ deg.) = (360\ deg.) * r \tag{3}$$

So, canceling out the r on both sides, we obtain

$$(2\pi\ radians) = 360\ deg. \tag{4}$$

or 1 radian = $\frac{360}{2\pi}\ deg.$ = 57.3 deg.

Stop a second now and see if you can make the following conversions:

1. 180 $deg.$ $\rightarrow$? radians

2. 60 $deg.$ $\rightarrow$? radians

3. 3π $radians$ $\rightarrow$? degrees

4. 2.5 $radians$ $\rightarrow$? degrees

Now that we have a bit of practice converting angles, lets talk about functions of angles: Draw yourself a triangle with base x , height y and hypotenuse r.

Now, we define the following:

$$\sin(\theta) = \frac{y}{r} \tag{5}$$

$$\cos(\theta) = \frac{x}{r} \tag{6}$$

Let's look at this triangle as being inscribed in the following circle of radius r:

So, we have that $x^2 + y^2 = r^2$. Or, dividing out by r, we have that

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \tag{7}$$

But, remember our definitions of $\cos(\theta)$ and $\sin(\theta)$! This leads us to

$$(\cos(\theta))^2 + (\sin(\theta))^2 = 1 \tag{8}$$

or in more common notation,

$$\cos^2(\theta) + \sin^2(\theta) = 1 \tag{9}$$

Keeping with the circle analogy, what happens to $\cos(\theta)$ and $\sin(\theta)$ if we add 360 deg., or 2π radians, to θ ? Or if we reflect θ to $-\theta$? Let's answer these questions pictorially:

What does the graph of $\sin(\theta)$ look like? What about $\cos(\theta)$?

One more formula for the road:

$$\sin(s+t) = \sin(s)\cos(t) + \cos(s)\sin(t) \tag{10}$$

We'll use this last formula to derive the form of the derivative of $\sin(x)$ in the next class!

Homework p.462-3 $1, 4, 21, 27, 37, 40$