# 21112 (Calculus 2) Lecture 2 - Indefinite Integrals

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Today, we introduce the idea of a general, or **indefinite**, integral. Instead of integrating between endpoints $a$ and $b$, we will find the function $\int f(x)dx$, such that $\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$.

A few notes:

$$\frac{d}{dx}x^m = mx^{m-1} \tag{1}$$

so, if we let $m := n+1$, then

$$\frac{d}{dx}x^{n+1} = (n+1)x^n \tag{2}$$

or better still,

$$x^n = \left(\frac{1}{n+1}\right)\frac{d}{dx}x^{n+1} = \frac{d}{dx}\left(\frac{1}{n+1}x^{n+1}\right) \tag{3}$$

so, by the definition of $\int f(x)dx$ given above, we have shown that for $n \neq -1$,

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{4}$$

Now, if $n = -1$, notice that $\frac{d}{dx}\ln(x) = x^{-1}$, so

$$\int \frac{1}{x}dx = \ln(x) \tag{5}$$

Before we go any further, notice that $\frac{d}{dx}C = 0$ for any constant $C$. So, we could add such a $C$ to any integral, and it would still be correct to call it 'the integral of $f(x)$'. Hence, we restate the above as such:

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C \tag{6}$$

$$\int \frac{1}{x}dx = \ln(x) + C \tag{7}$$

Finally, notice that $\frac{d}{dx}e^{ax} = ae^{ax}$, so by similar argument,

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + C \tag{8}$$

Now, since differentiation is linear, *i.e.* if we take $c_1, c_2$ real and $f(x), g(x)$ real functions, then

$$\frac{d}{dx}\left(c_1 f(x) + c_2 g(x)\right) = c_1\frac{d}{dx}f(x) + c_2\frac{d}{dx}g(x) \tag{9}$$

so, it follows directly that

$$\int (c_1 f(x) + c_2 g(x))\, dx = c_1 \int f(x)dx + c_2 \int g(x)dx \tag{10}$$

We are now ready to do some examples:

**Example 1**

**Find** $\int \left(x^2 + \frac{1}{4}x^{-2} + 2e^{-x}\right) dx$

ANSWER:

$$\int \left(x^2 + \frac{1}{4}x^{-2} + 2e^{-x}\right) dx = \int x^2 dx + \frac{1}{4}\int x^{-2}dx + 2\int e^{-x}dx$$
$$= \frac{1}{3}x^3 - \frac{1}{4}x^{-1} - 2e^{-x} + C$$

**Example 2**

A simulation of a factory's electricity usage shows a *rate of consumption* of $t - \frac{1}{t}$ units per hour after 1 a.m.. ($t$ is in hours.) If, at 1 a.m. the factory has already used up 10 units of electricity, then find the total electricity usage function. By this, we mean the **total** amount of units used.

ANSWER:

Let $U(t)$ denote consumption as a function of time $t$. Then, we have that $U'(t) = t - \frac{1}{t}$, $t \geq 1$, and $U(1) = 10$. So,

$$U(t) = \int t - \frac{1}{t}dt$$
$$= \frac{1}{2}t^2 - \ln(t) + C$$

but $10 = U(1) = \frac{1}{2}(1)^2 - \ln(1) + C = C + \frac{1}{2}$. So, $C = \frac{19}{2}$ and so $U(t) = \frac{1}{2}t^2 - \ln(t) + \frac{19}{2}$.

**Example 3**

A company has a marginal cost of $10x + e^{-2x}$ dollars/unit for units of $x$ and a fixed cost of 1000 dollars. What is the total cost ?

ANSWER

$Cost(x) = \int 10x + e^{-2x}dx = 5x^2 - \frac{1}{2}e^{-2x} + C$. But, $Cost(0) = 1000$, so $1000 = C(0) = 0 - \frac{1}{2} + C$, and it follows that $Cost(x) = 5x^2 - \frac{1}{2}e^{-2x} + 1000.5$.

**Homework**

Read over Examples $5-8$ in 6.1 of the text, and do problems $13, 23, 25, 32, 36, , 38, 39$ on $pp320 - 321$.