21112 (Calculus 2) Lecture 13 - Partial Differentiation

Albert Cohen

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Now that we have seen what multivariable functions look like, let us begin to differentiate them. But, just as we constructed these multivariable functions from $1-d$ slices, we will utilize similar ideas to differentiate these new functions.

1 $1-d$ derivative

Recall that we define $\frac{df}{dx}$ as follows:

$$\frac{df}{dx} := \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{1}$$

This *limit definition*, while impractical for computation, is the basis for all of the differentiation we do. It is *highly* recommended you review this formula and use it to calculate the derivatives of $f(x) = x, x^2$, and $\frac{1}{x}$.

2 Extension to $2-d$

What if we have a $2-d$ function $f(x, y)$, i.e a function that depends on 2 input variables? How can we construct "derivatives" for such a function? It turns out there are many ways to do so, but fundamentally, all such ways are derived from teh following 2 types:

$$f_x(x, y) = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h} \tag{2}$$

$$f_x(x, y) = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h} \tag{3}$$

Simply put, we hold one variable constant, and take the derivative with respect to the other variable.

With this definition in mind, we can formally calculate derivatives as we did in the $1-d$ case. However, for our case, we can employ all of our derivative rules from the $1-d$ case to the $2-d$ case as well.

3 Examples

Compute f_x, f_y for the following examples:

1. $f(x,y) = x^2 + y^2$
2. $f(x,y) = e^{x^2+y^2}$
3. $f(x,y) = x\sin(xy)$
4. $f(x,y) = \frac{y}{x}$
5. $f(x,y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$
6. $f(x,y) = \ln(xy^2)$
7. $f(x,y) = \frac{x+e^y}{x-e^y}$
8. $f(x,y) = \frac{x}{x+y}$

4 Homework

$p.403-4$,- $1, 4, 6, 8, 13, 19, 26$