21112 (Calculus 2) Lecture 12 - Intro to Multivariable Functions: Level Sets

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After having seen so much $1-d$ Calculus, we now enter the next and final stage of the course: Multivariable functions and the calculus that we can do with them. We begin by looking at how such functions can be constructed from $1-d$ curves.

How can we extend $x - y$ curves into the $x - y - z$ world ?

The beauty of Mathematics is that we build upon old topics instead of learning and dismissing them. Most problems of interest in the areas of Physics, Economics, Social Sciences, Natural Sciences and so on have more than one input variable affecting the outcome of the experiments people in those fields conduct. However, many times, they hold all but one variable constant (i.e. 'control groups' in medical experiments) and look at the affect of the isolated variable on the experiment.

As an example, let us consider the proposed model for temperature on a disc:

$$z = T(x, y) = x^2 + y^2 \tag{1}$$

for $\{(x, y) \in R^2 \mid x^2 + y^2 \leq 25\}$

What happens when we hold y constant at $y = 4$? How does the temperature vary as x varies? Well,

$$z = T(x, 4) = x^2 + 16 \tag{2}$$

How does this look in the $x - z$ plane ? What happens if we hold y constant at $y = 3$? $y = 2$? $y = 1$? $y = 0$?

Let's piece these all together!

What if we hold the x constant at the same values ? Does this give us any idea of what the function $z = x^2 + y^2$ should look like ?

Let's try another example. It is found that the amount of a drug remaining at time t that started with an initial dosage of y is

$$A(x, y) = ye^{-x} \tag{3}$$

Let's repeat the same method we used to visualize the previous function:

First, we hold y constant at the values $y = 1, 2, 3, 4$ and graph them on the $x - z$ coordinate axes. We now continue, but hold x constant at $x = 1, 2, 3, 4$. Let's draw these together:

How about if we take the function

$$z = \sqrt{x^2 + y^2} \tag{4}$$

What happens as we attempt to keep x constant, or y constant ?

Even better, let's hold z constant! What will these look like ? Let's go back and hold z constant for the previous examples. The resulting curves we get are called **level sets**.

Homework

Read Chapter 7.1, and look at the multivariable graphs they display. Also, draw the following level sets for $z = 1, 2, 3$:

1. $z = xy$
2. $z = xy^2$
3. $z = y\sec(x)$