

# 21112 (Calculus 2) Lecture 10 - Trigonometric functions - Episode 2: Attack of the Derivatives

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Now that we've derived the derivative for  $\sin(x)$  (with a little help,) we turn to evaluating more and more derivatives! No holds barred: product rule, chain rule, quotient rule.

Maybe some diagrams in this lecture.

With the help of a bonus point winner (hopefully :) ), we now know the following:

$$\frac{d}{dx} \cos(x) = -\sin(x) \quad (1)$$

So, in our collection of trigonometric derivatives, we have

1.  $\frac{d}{dx} \sin(x) = \cos(x)$
2.  $\frac{d}{dx} \cos(x) = -\sin(x)$

Now, there are more trig functions to deal with, namely

1.  $\tan(x)$
2.  $\sec(x)$
3.  $\csc(x)$

How do we find the derivatives of these ?

The answer lies in applying the derivative tricks we already know to the definitions of these functions - namely that they are all built from  $\cos(x)$  and  $\sin(x)$ .

$$1. \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$2. \sec(x) = \frac{1}{\cos(x)} = (\cos(x))^{-1}$$

$$3. \csc(x) = \frac{1}{\sin(x)} = (\sin(x))^{-1}$$

so,

$$\begin{aligned} \frac{d}{dx}(\tan(x)) &= \frac{d}{dx} \left( \frac{\sin(x)}{\cos(x)} \right) \\ &= \frac{\cos(x) \frac{d}{dx}(\sin(x)) - \sin(x) \frac{d}{dx}(\cos(x))}{(\cos(x))^2} \\ &= \frac{\cos^2(x) + \sin^2(x)}{(\cos(x))^2} \\ &= \frac{1}{(\cos(x))^2} \\ &= \left( \frac{1}{\cos(x)} \right)^2 \\ &= \sec^2(x) \end{aligned}$$

How about  $\frac{d}{dx} \sec(x)$ ?  $\frac{d}{dx} \csc(x)$ ?

$$\begin{aligned} \frac{d}{dx}(\sec(x)) &= \frac{d}{dx} \left( \frac{1}{\cos(x)} \right) \\ &= \frac{\cos(x) \frac{d}{dx}(1) - 1 \frac{d}{dx}(\cos(x))}{(\cos(x))^2} \\ &= \frac{\sin(x)}{(\cos(x))^2} \\ &= \frac{1}{\cos(x)} \frac{\sin(x)}{\cos(x)} \\ &= \sec(x) \tan(x) \end{aligned}$$

Could we use any other rules to do this ? Notice that  $\frac{d}{dx} \csc(x)$  is done exactly the same way.

Let's try some more fun ones!

Write down a method to differentiate each of the following:

1.  $\frac{d}{dx} \sin^2(x)$

2.  $\frac{d}{dx} e^{\sin^2(x)}$

3.  $\frac{d}{dx} \ln(\sin^2(x))$

4.  $\frac{d}{dx} e^x \sin(x)$

5.  $\frac{d}{dx} \cos(x) \tan(x)$

6.  $\frac{d}{dx} \frac{e^{\sin(x)}}{\cos(x)}$

7.  $\frac{d}{dx} \frac{\sin^2(x)-1}{\cos(x)}$

8.  $\frac{d}{dx} \frac{1}{2-\sin^2(x)}$

**Homework** Find the following derivatives:

1.  $\frac{d}{dx} e^{\sin(x)}$

2.  $\frac{d}{dx} \ln(\sin(x))$

3.  $\frac{d}{dx} \sin(x)e^{\sin(x)} - \sin^2(x)$

4.  $\frac{d}{dx} \sin(\sin(x))$