Department of Mathematical Sciences
Carnegie Mellon University

21-366 Random Graphs
Test 2

Name:____________________________________

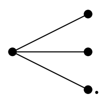
Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

Q1: (35pts)
Show that if 4 divides n and $np^4 \gg \log n$ then w.h.p. $G_{n,p}$ contains $n/4$ vertex disjoint copies of K_4 – the complete graph on 4 vertices.
Solution: Partition $[n]$ into 4 sets V_1, V_2, V_3, V_4 of size $n/4$. Because $\frac{n}{4}p \gg \log \frac{n}{4}$ there will w.h.p. be perfect matchings M_1, M_2 of the bipartite graphs induced by V_1, V_2 and V_3, V_4 respectively. Given these matchings, all edges not contained in a V_i are still unconditioned. We consider the bipartite graph H with vertices M_1, M_2 and an edge between $e \in M_1$ and $f \in M_2$ if $G_{n,p}$ completes e, f to a copy of K_4. This happens with probability p^4 and because $\frac{n}{4}p^4 \gg \log \frac{n}{4}$, H will w.h.p. contain a perfect matching that corresponds to $n/4$ vertex disjoint copies of K_4.

Q2: (35pts)
Show that if $p \geq \frac{10 \log n}{n}$ then w.h.p. it contains a cycle of length exactly $\lfloor n/2 \rfloor$.

Solution: We have $\frac{n}{2} p \geq 5 \log \frac{n}{2}$ and so w.h.p. $[\lfloor n/2 \rfloor]$ will contain a Hamilton cycle.

Q3: (30pts)
Show that if 4 divides n and $p \geq \frac{10 \log n}{n}$ then w.h.p. $G_{n,p}$ contains $n/4$ vertex disjoint copies of

$K_{1,3}$ 

Solution: Partition $[n]$ into 4 sets V_1, V_2, V_3, V_4 of size $n/4$. Because $\frac{n}{4}p \gg \log \frac{n}{4}$ there will w.h.p. be perfect matchings M_1, M_2, M_3 between V_1 and $V_i, i = 2, 3, 4$. These matchings define $n/4$ vertex disjoint copies of $K_{1,3}$.