Homework 6

6.7.20 Consider the modified greedy matching algorithm where you first choose a random vertex \( x \) and then choose a random edge \( \{x, y\} \) incident with \( x \). Show that applied to \( G_{n,m} \), with \( m = cn \), that w.h.p. it produces a matching of size \( \approx \left( \frac{1}{2} - \frac{\log(2 - e^{-2c})}{4c} \right) n \).
(If you can set up the associated differential equation, that will suffice.)

7.6.1 Let \( p = d/n \) where \( d \) is a positive constant. Let \( S \) be the set of vertices of degree at least \( \frac{2 \log n}{n \log \log n} \). Show that \( S \) is an independent set w.h.p.

7.6.9 Suppose that \( H \) is obtained from \( G_{n,1/2} \) by planting a clique \( C \) of size \( m = n^{1/2} \log n \) inside it. describe a polynomial time algorithm that w.h.p. finds \( C \). (Think that an adversary adds the clique without telling you where it is).
(How does adding the clique change the degree sequence?)