Name:______________________________

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Q1: (33pts)

Consider the Knapsack problem:

\[
\begin{align*}
\text{Maximise} & \quad p_1x_1 + p_2x_2 + \cdots + p_nx_n \\
\text{Subject to} & \quad w_1x_1 + w_2x_2 + \cdots + w_nx_n \leq W \\
& \quad x_1, x_2, \ldots, x_n \geq 0 \text{ and integer.}
\end{align*}
\]

Assuming that \( p_1/w_1 \geq p_2/w_2 \geq \cdots \geq p_n/w_n \) the Greedy Algorithm puts \( x_1 = \lfloor W/w_1 \rfloor, x_2 = \lfloor (W - w_1x_1)/w_2 \rfloor, x_3 = \lfloor (W - w_1x_1 - w_2x_2)/w_3 \rfloor \) and so on. Show that the value of the solution produced is always at least half the value of the optimal solution.
Q2: (33pts) A scout is going on a trip. She must select from a set of $n$ items. The items fall into $m$ types and $T_i$ is the set of items of type $i$. Item $j$ is of value $v_j$ for $j = 1, 2, \ldots, n$. There are some restrictions on what she can take:

1. She can take between $a_i$ and $b_i$ items of type $i$ for $i = 1, 2, \ldots, m$.

2. There is a list of pairs of items $L_1$ such that if $(j, k) \in L_1$ then she cannot take both of items $j$ and $k$.

3. There is a list of triples of items $L_2$ such that if $(i, j, k) \in L_2$ then if she takes both of items $i$ and $j$, then she must also take item $k$.

Construct an integer program that will solve the problem of maximising the total value of the items she can take.
Q3: (34pts) Find the optimal ordering strategy for the following inventory system. If you order an amount $Q$, it arrives immediately and the cost of the order is $A$. The inventory cost is $I$ per unit of inventory per unit of time. The demand per period, $t$ units of time after an order is $2(Q^{1/2} - t)$ and no stock-outs are allowed. You must re-order when the inventory reaches zero.