Department of Mathematical Sciences
Carnegie Mellon University
21-393 Operations Research II
Test 2

Name:______________________________

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Q1: (33pts) The playoff matrix $A$ of a two-person zero-sum game has $n$ rows and $n$ columns and is invertible. Let $1$ denote the $n$ dimensional column vector $(1,1,1,\ldots,1)^T$. Suppose that

$$1^T A^{-1} 1 \neq 0 \text{ and } A^{-1} 1 \geq 0 \text{ and } 1^T A^{-1} 1 \geq 0.$$  

Show, using its primal and dual Linear Programming formulations, that the game has value

$$V = \frac{1}{1^T A^{-1} 1}$$

and optimal row and column strategies

$$p = V 1^T A^{-1} \text{ and } q = V A^{-1} 1$$

respectively.
Q2: (33pts) Solve the following 2-person zero-sum games:

\[
\begin{bmatrix}
5 & 4 & 4 & 1 \\
6 & 5 & 5 & 2 \\
4 & 2 & 5 & 5 \\
6 & 5 & 2 & 5 \\
\end{bmatrix}
\quad
\begin{bmatrix}
2 & 2 & 0 & -1 \\
4 & 3 & 0 & -1 \\
3 & 2 & 1 & -1 \\
1 & 1 & -1 & 1 \\
\end{bmatrix}
\]
Q3: (33pts) There are 3 assets with data given below:

\[
V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/2 \\ 0 & 1/2 & 1 \end{bmatrix}, \quad \bar{r} = \begin{bmatrix} 3 \\ 5 \\ .8 \end{bmatrix}
\]

Find 2 efficient funds \( F_1, F_2 \) for which every other efficient portfolio can be expressed as a linear combination \( \alpha F_1 + (1 - \alpha) F_2 \).