Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
(a) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

$$\begin{array}{lccccccc} \text{maximise} & 3x_1 & + & 7x_2 & + & 12x_3 & & \\ \text{subject to} & & & & & & & \\ & 2x_1 & + & 3x_2 & + & 5x_3 & \leq & 12 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Your answer should consist of a table.
(b) Using the answer to part (a), solve the following problem:

$$\begin{array}{lccccccc} \text{minimise} & 2x_1 & + & 3x_2 & + & 5x_3 & & \\ \text{subject to} & & & & & & & \\ & 3x_1 & + & 7x_2 & + & 12x_3 & \geq & 20 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

(This does not require any new computations!)

Q2: (33pts) A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i,j)$, where the $c(i,j)$ are given in the matrix below. The one period discount factor α is 1/2.
The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 3, \pi(3) = 2.$$

Evaluate this policy. Is it optimal? If not find an improved policy.
YOU DO NOT NEED TO EVALUATE THIS NEW POLICY OR FIND AN OPTIMAL STRATEGY.
The matrix of costs is

$$\begin{bmatrix} 7 & 3 & 1 \\ 4 & 2 & 7 \\ 1 & 8 & 2 \end{bmatrix}$$

Q3: (34pts) Yuckdonald's is considering opening a series of restaurants along Quaint Valley Highway (QVH). The n possible locations are along a straight line and the distances of these locations from the start of QVH are, in miles and in increasing order, $d_1, d_2, \ldots, d_n$. The constraints are as follows:

- At each location, Yuckdonald's can open at most one restaurant. The expected profit from opening a restaurant at location i is p_i, where $p_i > 0$ for $i = 1, 2, \ldots, n$.
- Any two restaurants should be at least m miles apart, where m is a positive integer.

Describe a Dynamic Programming solution to the problem of maximising the expected profit subject to the above constraints.
[Hint: Break up a stick of length n.]