Department of Mathematical Sciences
Carnegie Mellon University
21-393 Operations Research II
Test 1

Name:_______________________________

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Q1: (33pts)
(a) Solve the following knapsack problem, writing the results of the dynamic programming recursion in a table. You will not score any points for just writing down the answer:

maximise \[ 3x_1 + 8x_2 + 15x_3 \]
subject to \[ 2x_1 + 3x_2 + 5x_3 \leq 10 \]
\[ x_1, x_2, x_3 \geq 0 \text{ and integer.} \]

Your answer should consist of a table.
(b) Using the answer to part (a), solve the following problem:

minimise \[ 2x_1 + 3x_2 + 5x_3 \]
subject to \[ 3x_1 + 8x_2 + 15x_3 \geq 22 \]
\[ x_1, x_2, x_3 \geq 0 \text{ and integer.} \]

(This does not require any new computations!)
Q2: **(33pts)** A system can be in 3 states 1, 2, 3 and the cost of moving from state $i$ to state $j$ in one period is $c(i, j)$, where the $c(i, j)$ are given in the matrix below. The one period discount factor $\alpha$ is 1/2.

The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 1, \pi(3) = 2.$$ 

Evaluate this policy. Is it optimal? If not find an improved policy.

**YOU DO NOT NEED TO EVALUATE THIS NEW POLICY OR FIND AN OPTIMAL STRATEGY.**

The matrix of costs is

$$
\begin{bmatrix}
6 & 2 & 1 \\
4 & 2 & 6 \\
1 & 6 & 2
\end{bmatrix}
$$
Q3: (33 pts) Dan Dare is flying his spaceship from Agmon to Zoron along the brand new Inter-Galactic Super Space Highway. The spaceship runs on Sillium fuel and there are $n$ places along the way to stop and purchase fuel. The price at stop $i$ is $p_i$ per gallon and has quality $q_i$. This means that each gallon will take the spaceship $q_i$ parsecs along its journey. The spaceship has fuel capacity $T$ and starts out with a full tank. Each re-fuelling costs $f$ in terms of fees for entering the fuelling station. The distance from station $i$ to station $j$ is $d_{i,j}$ parsecs. Assume that the start Agmon is at station 0 and the finish Zoron is at station $n + 1$. Assume also that the quality of the fuel in the tank at the start is $q_0$ and that adding $x$ gallons of fuel with quality $q$ to a tank with $y$ gallons of quality $r$ produces a tank with $x + y$ gallons of quality $\frac{rq+yr}{x+y}$.

Dan wishes to minimise the cost of the journey. Formulate the problem as a Dynamic Program.