Optimizing Hyperloop Track Construction

Creating a High-Speed Network to Connect the 25 Largest Cities in the United States

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Chapter 1

Background

For this project, we decided to investigate a hyperloop system. [4] The first mention of this new form of transportation came from Elon Musk in 2012 and involves a sealed tube (or an entire system of tubes) that travels with minimum air resistance. This lack of friction allows pods to move at extremely fast speeds while keeping the travel efficient. Since the project was only mentioned in 2012, there hasnt been very much movement by developers to create any hyperloops, but we found the overall idea quite interesting. We could have done an optimization problem on current transportation, such as the Pittsburgh bus system or flight patterns across the country, but we wanted to do something different from the norm.

One theoretical concept proposed in 2013, the Hyperloop Alpha, would have been a hyperloop between Los Angeles and San Francisco. This 350 mile path that would be completed in approximately 35 minutes could cut down the normal driving and flying time to 1/12 and 1/3, respectively. This specific system would also be six billion dollars for a passenger transport (with prices increasing for a larger tube for passengers and freight transport). There are a lot of factors that could be a concern with this project, including the sophistication of the technology, the need for different materials for different climates, and the high prices to build the pods and tracks (not counting how expensive it is to run a hyperloop). For this overall project we made a few major assumptions. We assumed that any kinks in the technology were resolved and that the limit of cost was not an issue. We also decided to simplify the cross-continental differences by making all the prices constant per mile, instead of having to account for building through the mountains, desert, or waterways. This left us with the plan to make a simple map of hyperloops.

Chapter 2

Problem

After looking into the background of what a Hyperloop system is, we decided we wanted to build a hypothetical map of railways between major cities across the United States. As mentioned earlier, actual hyperloops could be used for both passenger transport and cargo transport. We decided to focus solely on passenger movement from city to city. We wanted to choose a correct number of paths between these cities to maximize the number of passengers from city to city, while minimizing the overall cost. We assumed that passengers would want to choose our method of transportation if our price levels are competitive enough, but we, as a company, want to maximize our profits to keep the business afloat. We chose the top 25 metropolitan cities across the US for our project to lessen the number of paths needed. To solve the actual optimization problem, we used integer programming, minimum spanning tree, Dijkstras algorithm, and linear programming. These were used together to optimize the overall map (picking the best map between all 25 cities) and the number of passengers (i.e. the number of pods) to and from each city.

Chapter 3

Initial Model

We selected the 25 largest cities in the united states by metropolitan area population (so including suburbs) [5] to connect with our network (see figure 3.1).

Define values:

- n is the number of cities we are including.
- The cities are arbitrarily numbered $1...n$
- each possible edge between two cities is defined as a (non-ordered) pair of cities ($\binom{n}{2}$ in total)
- each possible path between two cities is defined as an (ordered) list of cities that does not repeat entries ($\sum_{l=2}^{n} \frac{n!}{(n-l)!}$ in total)
- We say, for each edge e and path q, $e \in q$ iff the two cities connected by e are adjacent in the list of cities q.
- For each possible edge e:
  - c_e is the cost of building e.
  - p_e: Price we are charging for travel along e. Estimated from airline prices.
  - t_e: Length of time it takes to traverse e by hyperloop. Estimated by physical distance.
  - $X_e \equiv (1 : \text{edge } e \text{ exists in our graph}, 0 : \text{otherwise})$
- For each possible path q:
  - $|q|$: the number of cities in q
  - endpoints(q) is a set of two numbers for the cities that form the endpoints of q (non-ordered)
  - length(q) is the sum of lengths of the edges in q, equal to $\sum_{e \in q} t_e$.

- $Y_q \equiv (1 : \text{path } q \text{ exists in our graph and is the shortest path between its nodes}, 0 : \text{otherwise})$

- P is profit (value we are trying to maximize)
- for each pair of cities i, j: $a_{i,j}$ is an estimation of demand for travel between those cities–in our case, the number of people who travel yearly between those cities by air.

Objective equation:

$$P = \sum_{q \in Q} r_q Y_q - \sum_{e \in E} c_e X_e$$

where r_q, the revenue we think we will get from path q, is calculated as such:

$$r_q = (a_q - \lambda_1 \text{length}(q) - \lambda_2 |q|) \left(\sum_{e \in q} p_e \right)$$

λ_1 and λ_2 are values that can be tuned. λ_1 represents the number of customers we think we'll lose per minute of travel time, and λ_2 represents the number of customers we think we'll lose per edge.
Constraints:

$$\forall q : \forall e \in q : X_e \geq Y_q$$

If a path exists, each edge in it must exist

$$\forall \text{ cities } a \neq b : \left(\sum_{\substack{q \\ \text{endpoints}(q)=(a,b)}} Y_q \right) \leq 1$$

There is at most one shortest path between every pair of nodes

$\forall$ paths q, r s.t. $\text{length}(q) < \text{length}(r)$ and $\text{endpoints}(q) = \text{endpoints}(r) : Y_q \geq Y_r$

If a path is the shortest path, another strictly shorter path between the same endpoints must also be the shortest path.

Figure 3.1: 25 largest cities

Chapter 4

Simplified Model

It was decided that the initial ILP would be prohibitively computationally expensive to solve, so several simplifications were made to reduce it to a more solvable problem. First, we opted to leave the ILP framework, and instead iterate through a set of reasonable networks and evaluate and compare the profits of each one. Each network considered was built on two sets of connections along the east and west coasts that were deemed necessary (4.1).

We then defined a set of reasonable intermediate connections, and the family of networks considered was defined as all possible ways to add some subset of these to the coastal connections (4.2).

We eliminated every network that was not connected, reasoning that creating a national hyperloop network implies an objective of connecting all major cities. We also eliminated all networks with more than five edges going into a single node, assuming that creating many connections between hyperloop tracks in one city would be difficult and expensive. In order to evaluate the revenue of each reasonable network, we first calculated the shortest path between each pair of cities using a modified Dijkstra algorithm. The modification was the assumption that customers would opt to fly instead if the shortest path contained more than three edges (two connections). If no three-edge path was found, the cities were treated as unreachable. Finally, the revenue was equal to airline revenue between those two cities. The cost of each network was the sum of approximate costs of the edges that exist in that network.

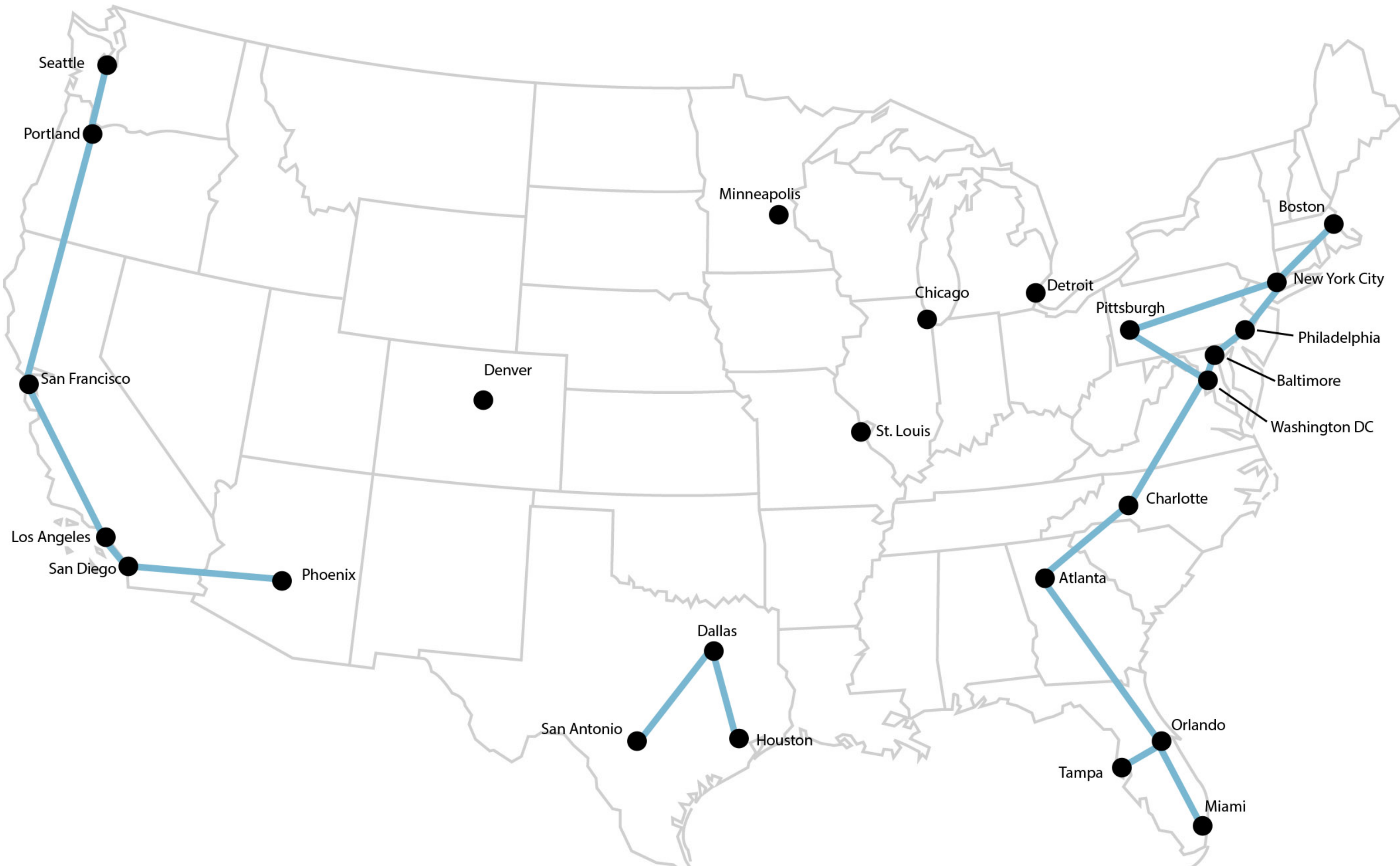

Figure 4.1: Necessary coastal connections

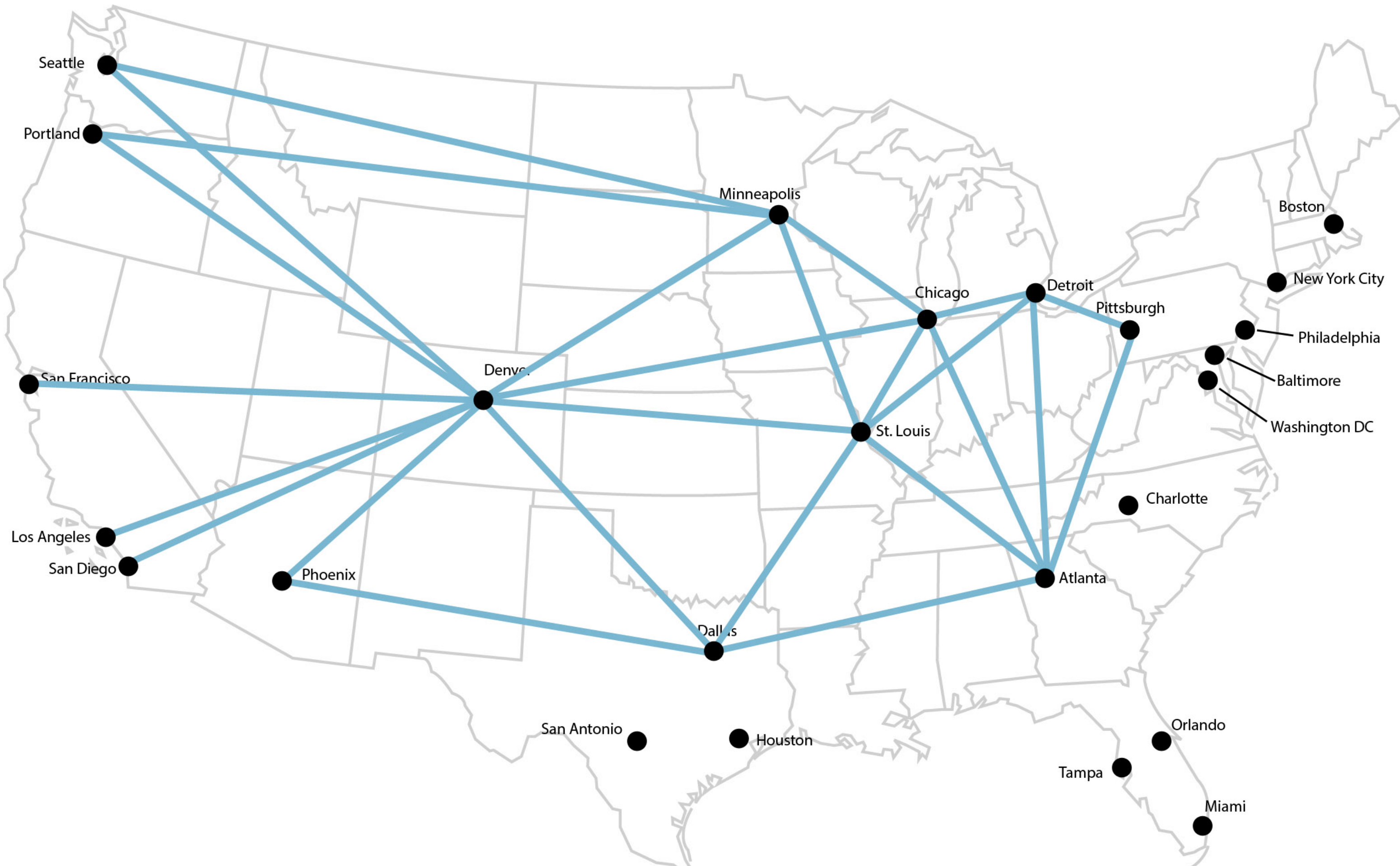

Figure 4.2: Possible intermediate connections

Chapter 5

Data

5.1 Distance Data

The distance between the 25 cities was determined from a flight dataset from the Department of Transportations Website [3].

5.2 Pricing Data

As the hyperloops main advantage is its speed, the goal was to take away market share from US domestic airlines. Consequently, our pricing plan should be cheaper than that of a plane. Likewise, we aim to target train users who have a higher willingness to pay for a faster train since pricing at the same price of trains would likely be infeasible from a profit perspective. The team was able to obtain some useful flight pricing data for 2017-2018. The standard industry metrics used here are Cost Per Available Seat Mile (CASM) and Revenue Per Available Seat Mile (RASM). These metrics are ideal because they are adjusted for the different business models (i.e a low cost airline like Frontier vs standard airlines like American). Figure 5.1 below is from a detailed economic analysis of domestic flights by the management consulting company Oliver Wyman [1]. With this table we will assume that pricing below the lowest RASM (Frontier Airlines) of 9.4 cents/mile would entice new customers to switch to Hyperloop. Thus our upper bound on price is \$0.094. For our model we would be pricing at \$0.09/mile.

5.3 Cost Data

: The cost per mile estimates of building the tubes (tracks) were obtained from Elon Musks estimates on investopedia website at \$15,400,000.00/mile [2].

Hyperloop Fee = (Number of Miles)*(price/mile) = (606 miles) *(\$0.09) = \$54.54

Exhibit 49: System RASM/CASM by Airline, Q2 2017

AIRLINE	RASM (CENTS)	CASM (CENTS)	MARGIN (CENTS)	MARGIN %
Alaska/Virgin America	13.5	10.0	3.4	25.5%
Hawaiian	14.2	11.1	3.2	22.2%
Southwest	14.3	11.2	3.1	21.8%
Frontier	9.4	7.4	2.1	21.7%
Allegiant	10.9	8.7	2.2	20.0%
JetBlue	12.9	10.4	2.5	19.2%
Spirit	9.6	7.8	1.8	18.8%
Delta	16.4	13.3	3.1	18.8%
United	14.8	12.7	2.1	14.0%
American	15.5	13.3	2.1	13.8%

Source: PlaneStats.com

Figure 5.1: CASM and RASM for 2017 of Domestic Airlines

ORIGIN_AIRPORT	ORIGIN_C	ORIGIN_CITY_NA	DEST_AIR	DEST_AIRPORT	DEST_CITY_MARKE	DEST_CITY_NAME	FLIGHTS	DISTANCE
1402702	34027	West Palm Beac	12953	1295302	31703	New York, NY	1	1035
1247803	31703	New York, NY	13204	1320402	31454	Orlando, FL	1	944
1169704	32467	Fort Lauderdale,	11433	1143302	31295	Detroit, MI	1	1127
1129202	30325	Denver, CO	11433	1143302	31295	Detroit, MI	1	1123
1039705	30397	Atlanta, GA	11995	1199502	31995	Greensboro/High	1	306
1289204	32575	Los Angeles, CA	14831	1483104	32457	San Jose, CA	1	308
1393004	30977	Chicago, IL	10397	1039705	30397	Atlanta, GA	1	606
1039705	30397	Atlanta, GA	12278	1227803	30928	Wichita, KS	1	782
1247803	31703	New York, NY	13204	1320402	31454	Orlando, FL	1	944
1039705	30397	Atlanta, GA	11618	1161802	31703	Newark, NJ	1	746
1143302	31295	Detroit, MI	14107	1410702	30466	Phoenix, AZ	1	1671

Figure 5.2: Data Set of flight data From the Department of Transportation

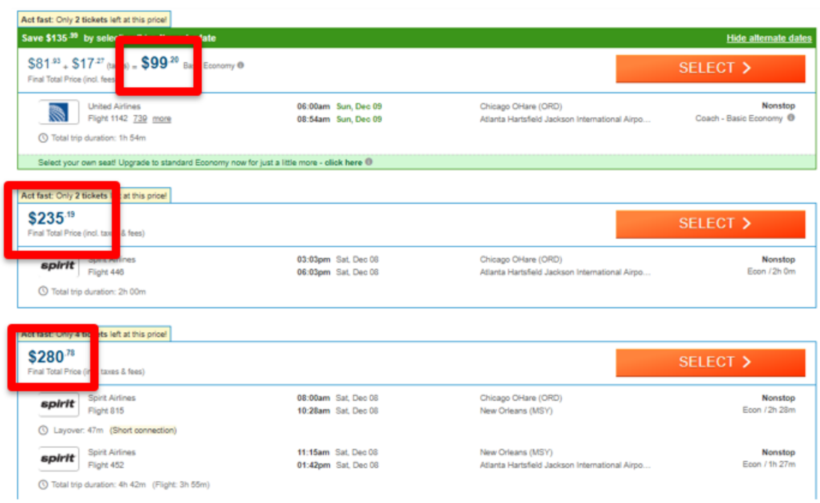

Figure 5.3: Prices of Flights from Chicago to Atlanta from CheapOAir.com

Chapter 6

Results

In order to determine the effectiveness of our method, we needed a network to compare our results against. A natural choice for a connected network that minimizes cost is the minimum spanning tree of the network (6.1).

This network uses 6,495 miles of track at a total cost of $97,425,000,000. Using the same method for evaluating the revenues of a network as we used in the integer program we estimate that the revenues over a 10 year period for this network will be $110,119,340,880, which leaves profits of $12,694,340,880. Compare this result with the network selected by our integer program (6.2).

This network uses 11,995 miles of track at a total cost of $179,925,000,000. Using the method of evaluating the revenue of a network from our integer program we estimate that the revenues over a 10 year period for this network will be $399,094,369,920, which leaves profits of $219,169,369,920. Clearly using our method has substantially increased the predicted profitability of a hyperloop system. There is still one more step to complete before we have the final results. We now need to use and integer program to balance the flow of traffic into and out of each city. To do this we apply the following integer program to our networks:

Linear Program

$$\max_{x} \sum_{p=1 \ldots \mathrm{P}} MileCost * Distance_p * x_p$$

$$s.t. \quad \sum_{i \in \{\text{Paths into City k}\}} x_i - \sum_{o \in \{\text{Paths out of City k}\}} x_o = 0 \quad \text{for all } k \in 1\ldots 25$$

$$0 \leq x_p \leq Demand_p \quad \text{for all } p \in 1 \ldots P$$

where P is the number of paths in the network and x_p is the demand across path p

The linear program simply aims to maximize the total revenue (at this point we can ignore the cost of the tracks because those are fixed costs now that the networks are decided), subject to the constraint that the total passenger coming into the city from all other sources is equal to the total passengers leaving the city to all other cities. In addition, between any two cities the number of passengers is limited by the demands from the flight data. After applying this linear program to MST and the network from the integer program the profits after ten years are \$602,322,480 and \$187,511,883,072. The demands across all of the links after applying the linear program is depicted in 6.3.

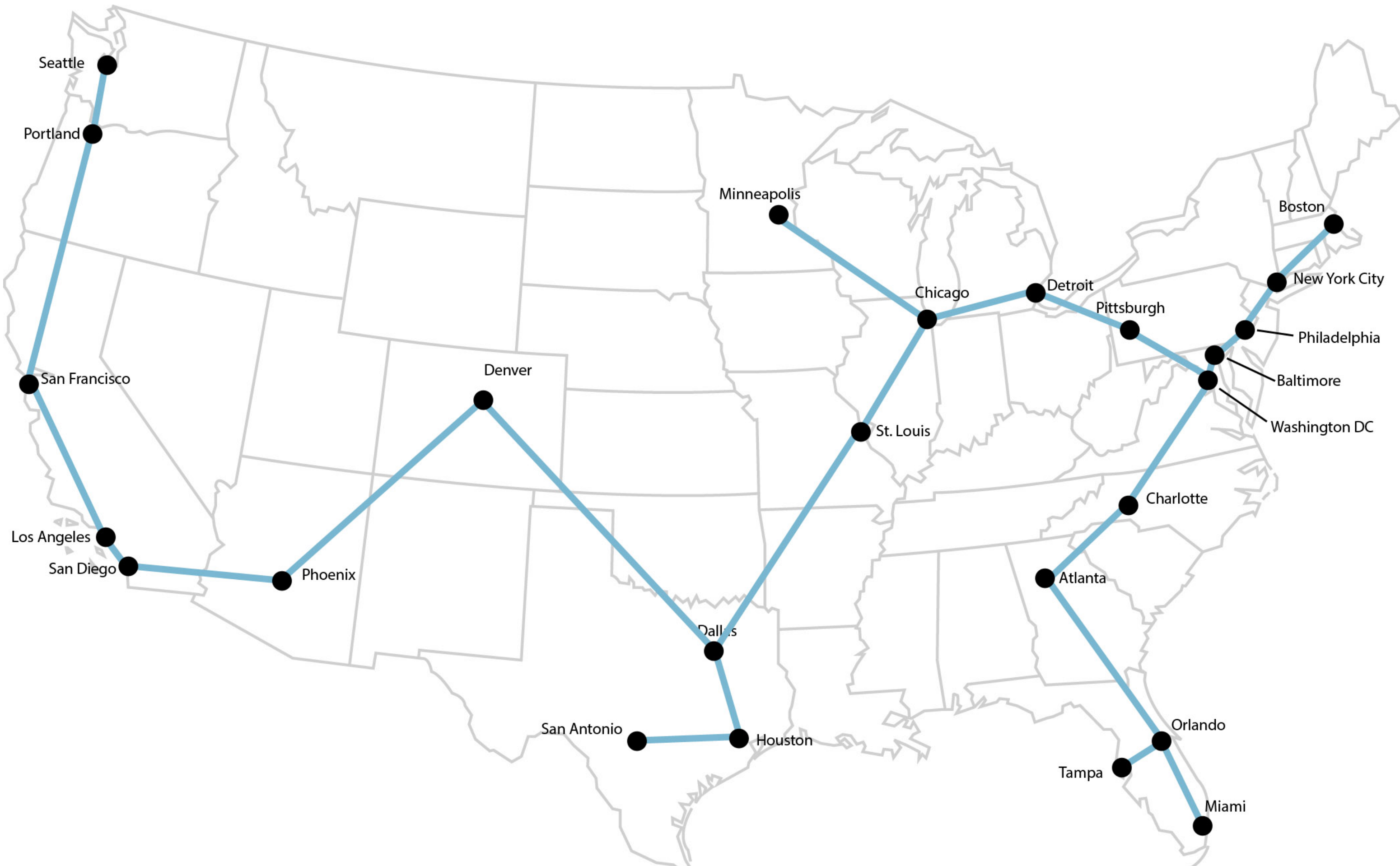

Figure 6.1: Minimum spanning tree

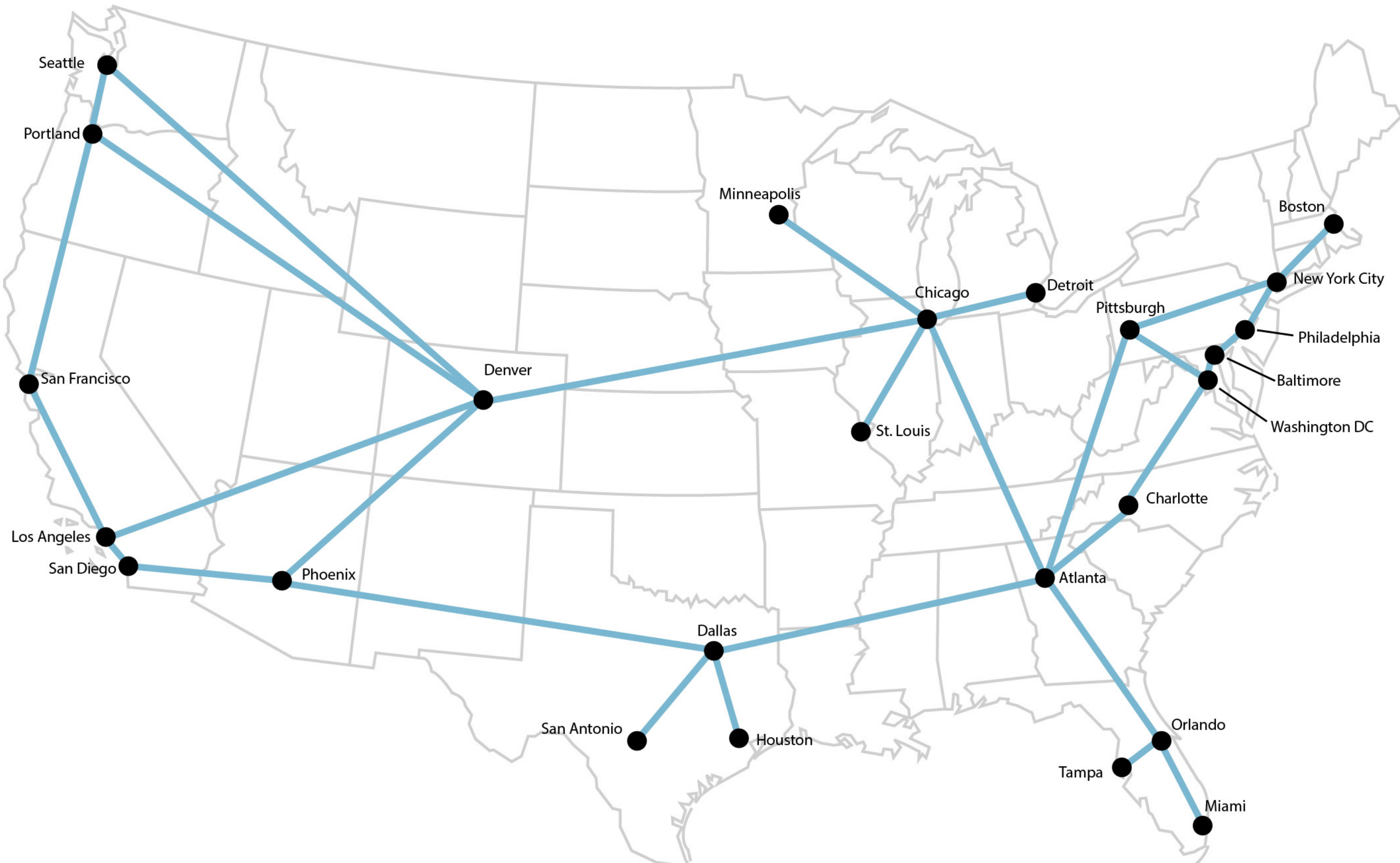

Figure 6.2: Our algorithm's output

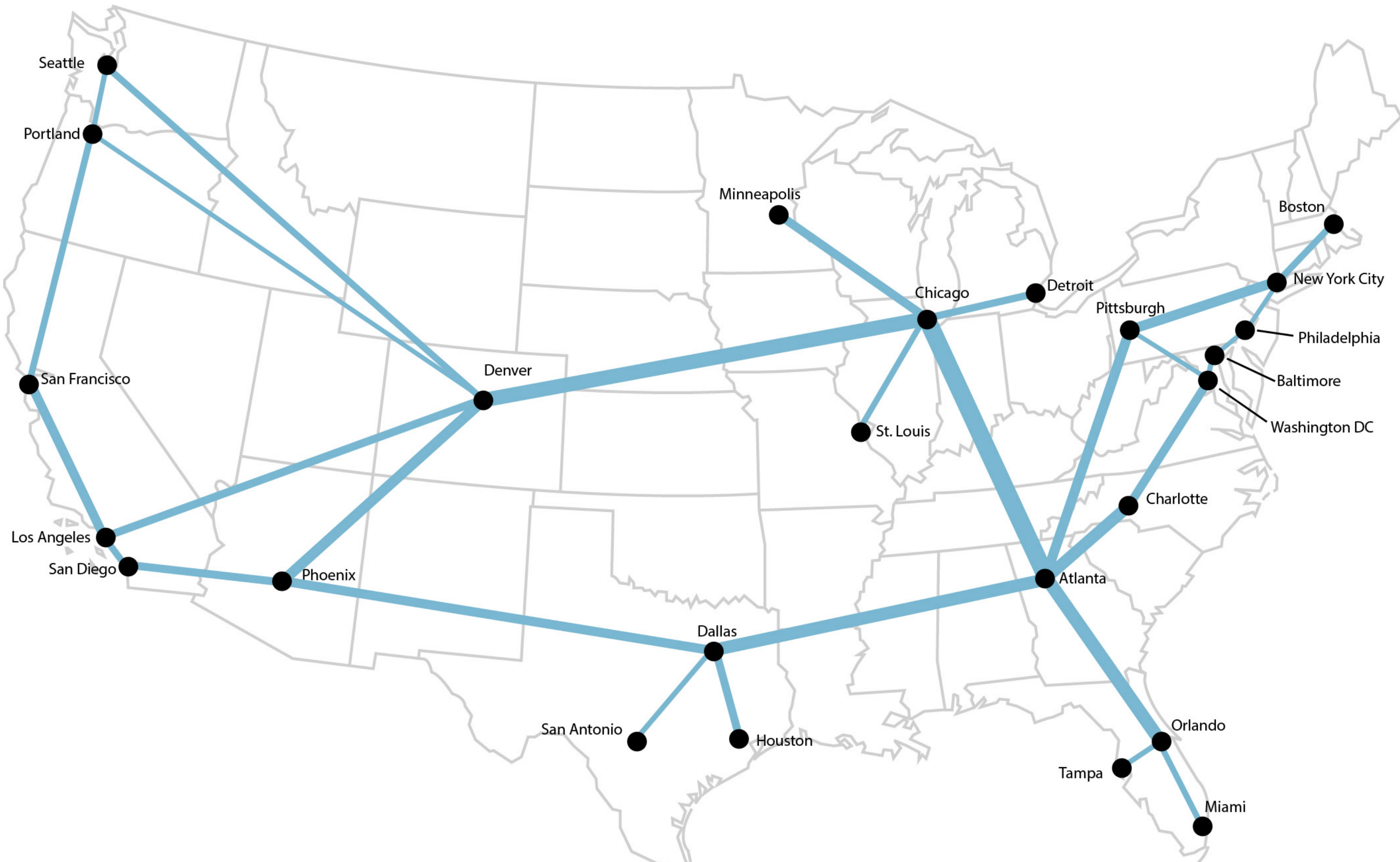

Figure 6.3: Our algorithm's output

Chapter 7

Possible Further Research

As stated before, while designing this model we had to make many assumptions to conform to our time limit and abilities. In order to obtain a more accurate solution to our problem, one would look into different types of pods with each accommodating a different number of people in order to to more closely mirror real life considerations. We additionally made quite a few assumptions about the fixed and variable costs, as well as competition and demand, that should be examined closer. These are some of the biggest factors that transportation companies look into and most likely would have an effect on the resulting model. Further research would not only include the modifications to the current problem stated above, but also would examine more complex and different types of networks. For example, one could look at the complex network of airlines and flight paths which are not fixed and are affected by even more constraints like weather. A similar network design process could be used to plan where public places such as gas stations and banks should be located throughout cities in order to address demand efficiently. On a larger scale, this linear programming approach could be used to determine where to place farms and electrical plants that require optimal networks for manufacturing, inventory as well as distribution.

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