Bus Routine Profit Maximization Problem

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Introduction

For students at Carnegie Mellon University, the buses 61A, 61B, 61C, and 61D are most commonly used in our everyday life not only because they all have stops at Carnegie Mellon University, but they cover districts like Squirrel Hill, Oakland, and Downtown, where most students live and hang out. These four types of buses are run by the Pittsburgh Port Authority. 61A and 61B cover routes from Braddock Hills Shopping Center to Downtown, 61C covers routes from McKeesport Transportation Center to Downtown, and 61D covers routes from Waterfront to Downtown. Usually, students take either 61A/61B, or 61C/61D because 61A and 61B go on almost the same routes and 61C and 61D have almost the same routes. For example, if a student wants to go see a movie at Waterfront, then he/she could take either 61C or 61D to get to the cinema directly, and if a student lives at Squirrel Hill, he/she could take either 61A or 61B to go to school. After doing some research, we found out that the Port Authority's operating cost per passenger is $5.79 and each passenger pays a transfer fee of $2.75 for each ride, regardless of the length of the ride. And after conducting surveys around CMU, we are positive to say that the bus service provided by Pittsburgh Port Authority is very trustworthy and recognized by CMU students, and that is why our team find it even more urgent to solve some problems that the Pittsburgh Port Authority is facing with these buses.

Understanding the current problems

First, let us take a closer look at the schedule of these four types of bus. Each schedule contains the inbound and outbound routes of the bus. Usually, inbound means the bus's final destination is Downtown Pittsburgh and outbound means the bus goes from Downtown Pittsburgh to other districts. Each route contains around 10 stops, and it takes around 40 minutes for 61A and 61B to finish one schedule and 50 minutes for 61C and 61D to finish one schedule. Sample schedules of the four buses are displayed in the following page [1][2].

61A NORTH BRADDOCK

MONDAY THROUGH FRIDAY SERVICE

To Downtown Pittsburgh

Braddock Hills Shopping Center Propel School	Braddock 6th St at Braddock Ave	Braddock 7th St at Braddock Ave	Braddock Verona St at Mapleview Dr	North Braddock Hawkins Ave at 4th St	Swissvale Woodstock Ave opp. Swissvale Sta Steps	Swissvale Noble St opp. Washington St	Wilkinsburg Coal St past Walnut St	Wilkinsburg Rowland Connector near Kelly Ave	Regent Square Forbes Ave past S Braddock Ave	Squirrel Hill Forbes Ave past Murray Ave	Oakland Forbes Ave at S Craig St	Oakland Fifth Ave opp. Atwood St	Downtown Fifth Ave at Wood St
....	....	4:56	4:57	5:01	5:04	5:07	5:12	5:17	5:20	5:24	5:28	5:32	5:45
....	....	5:17	5:18	5:22	5:25	5:28	5:34	5:40	5:44	5:49	5:55	6:00	6:15
....	....	5:47	5:48	5:52	5:55	5:58	6:04	6:10	6:14	6:19	6:25	6:30	6:45
....	....	5:57	5:58	6:03	6:07	6:10	6:16	6:23	6:29	6:35	6:43	6:49	7:07
....	....	6:12	6:13	6:18	6:22	6:25	6:31	6:38	6:44	6:50	6:58	7:04	7:22
6:21	6:26	....	6:28	6:33	6:37	6:40	6:46	6:53	6:59	7:05	7:13	7:19	7:37
....	....	6:42	6:43	6:48	6:52	6:55	7:01	7:08	7:14	7:20	7:28	7:34	7:52
....	....	6:57	6:58	7:03	7:07	7:10	7:16	7:23	7:29	7:35	7:43	7:49	8:07
7:06	7:11	....	7:13	7:18	7:22	7:25	7:31	7:38	7:44	7:50	7:58	8:04	8:22
....	....	7:27	7:28	7:33	7:37	7:40	7:46	7:53	7:59	8:05	8:13	8:19	8:37
7:36	7:41	....	7:43	7:48	7:52	7:55	8:01	8:08	8:14	8:20	8:28	8:34	8:52
....	....	7:58	7:59	8:03	8:07	8:10	8:16	8:23	8:29	8:35	8:43	8:49	9:07
8:07	8:12	....	8:14	8:18	8:22	8:25	8:31	8:38	8:44	8:50	8:58	9:04	9:22
8:22	8:27	....	8:29	8:33	8:37	8:40	8:46	8:53	8:59	9:05	9:13	9:19	9:37
8:38	8:43	....	8:45	8:49	8:53	8:56	9:02	9:09	9:15	9:21	9:29	9:35	9:53
....	....	9:10	9:11	9:15	9:19	9:22	9:28	9:34	9:37	9:42	9:49	9:55	10:13
9:24	9:29	....	9:31	9:35	9:39	9:42	9:48	9:54	9:57	10:02	10:09	10:15	10:33
....	....	9:50	9:51	9:55	9:59	10:02	10:08	10:14	10:17	10:22	10:29	10:35	10:53
10:04	10:09	....	10:11	10:15	10:19	10:22	10:28	10:34	10:37	10:42	10:49	10:55	11:13
10:24	10:29	....	10:31	10:35	10:39	10:42	10:48	10:54	10:57	11:02	11:09	11:15	11:33
....	....	10:50	10:51	10:55	10:59	11:02	11:08	11:14	11:17	11:22	11:29	11:35	11:53
....	....	11:10	11:11	11:15	11:19	11:22	11:28	11:34	11:37	11:42	11:49	11:55	**12:13**
....	....	11:30	11:31	11:35	11:39	11:42	11:48	11:54	11:57	**12:02**	**12:09**	**12:15**	**12:33**
11:44	11:49	....	11:51	11:55	11:59	**12:02**	**12:08**	**12:14**	**12:17**	**12:22**	**12:29**	**12:35**	**12:53**
....	....	**12:10**	**12:11**	**12:15**	**12:19**	**12:22**	**12:28**	**12:34**	**12:37**	**12:42**	**12:49**	**12:55**	**1:13**
....	....	**12:30**	**12:31**	**12:35**	**12:39**	**12:42**	**12:48**	**12:54**	**12:57**	**1:02**	**1:09**	**1:15**	**1:33**
....	....	**12:50**	**12:51**	**12:55**	**12:59**	**1:02**	**1:08**	**1:14**	**1:17**	**1:22**	**1:29**	**1:35**	**1:53**

To Oakland - Squirrel Hill - North Braddock

Downtown Fifth Ave at Wood St	Oakland Forbes Ave at Atwood St	Carnegie Museum Forbes Ave opp. S Craig St	Squirrel Hill Forbes Ave at Murray Ave	Regent Square Forbes Ave at East End Ave	Wilkinsburg Rowland Connector at Pennwood Ave	Edgewood Swissvale Ave at Coal St	Swissvale Noble St at Washington St	Swissvale Woodstock Ave at Swissvale Sta Steps	North Braddock Hawkins Ave at 4th St	Braddock Braddock Ave at Library St	Braddock Braddock Ave at 6th St	Braddock Hills Shopping Center Giant Eagle
5:45	5:56	5:59	6:04	6:09	6:12	6:18	6:22	6:25	6:28	6:32	6:33	6:39
6:15	6:26	6:29	6:34	6:39	6:42	6:48	6:52	6:55	6:58	7:02	7:03	7:09
6:45	6:56	6:59	7:04	7:09	7:12	7:18	7:22	7:25	7:28	7:32	7:33	7:39
7:07	7:23	7:26	7:32	7:37	7:40	7:47	7:51	7:54	7:57	8:02	8:03	8:09
7:22	7:38	7:41	7:47	7:52	7:55	8:02	8:06	8:09	8:12	8:17	8:18	8:24
7:37	7:53	7:56	8:02	8:07	8:10	8:17	8:21	8:24	8:27	8:32	8:33	....
7:52	8:08	8:11	8:17	8:22	8:25	8:32	8:36	8:39	8:42	8:47	8:48	8:54
8:07	8:23	8:26	8:32	8:37	8:40	8:47	8:51	8:54	8:57	9:02	9:03	9:09
8:22	8:38	8:41	8:47	8:52	8:55	9:02	9:06	9:09	9:12	9:17	9:18	....
8:37	8:53	8:56	9:02	9:07	9:10	9:17	9:21	9:24	9:27	9:32	9:33	9:39
8:52	9:08	9:11	9:17	9:22	9:25	9:32	9:36	9:39	9:42	9:47	9:48	....
9:07	9:21	9:24	9:31	9:37	9:40	9:46	9:50	9:53	9:56	10:01	10:02	10:08
9:22	9:36	9:39	9:46	9:52	9:55	10:01	10:05	10:08	10:11	10:16	10:17	....
9:37	9:51	9:54	10:01	10:07	10:10	10:16	10:20	10:23	10:26	10:31	10:32	....
9:53	10:07	10:10	10:17	10:23	10:26	10:32	10:36	10:39	10:42	10:47	10:48	....
10:13	10:27	10:30	10:37	10:43	10:46	10:52	10:56	10:59	11:02	11:07	11:08	11:14
10:33	10:47	10:50	10:57	11:03	11:06	11:12	11:16	11:19	11:22	11:27	11:28	....
10:53	11:07	11:10	11:17	11:23	11:26	11:32	11:36	11:39	11:42	11:47	11:48	11:54
11:13	11:27	11:30	11:37	11:43	11:46	11:52	11:56	11:59	**12:02**	**12:07**	**12:08**	....
11:33	11:47	11:50	11:57	**12:03**	**12:06**	**12:12**	**12:16**	**12:19**	**12:22**	**12:27**	**12:28**	....
11:53	**12:07**	**12:10**	**12:17**	**12:23**	**12:26**	**12:32**	**12:36**	**12:39**	**12:42**	**12:47**	**12:48**	**12:54**
12:13	**12:27**	**12:30**	**12:37**	**12:43**	**12:46**	**12:52**	**12:56**	**12:59**	**1:02**	**1:07**	**1:08**	**1:14**
12:33	**12:47**	**12:50**	**12:57**	**1:03**	**1:06**	**1:12**	**1:16**	**1:19**	**1:22**	**1:27**	**1:28**	**1:34**
12:53	**1:07**	**1:10**	**1:17**	**1:23**	**1:26**	**1:32**	**1:36**	**1:39**	**1:42**	**1:47**	**1:48**	....
1:13	**1:27**	**1:30**	**1:37**	**1:43**	**1:46**	**1:52**	**1:56**	**1:59**	**2:02**	**2:07**	**2:08**	**2:14**
1:33	**1:47**	**1:50**	**1:57**	**2:03**	**2:06**	**2:12**	**2:16**	**2:19**	**2:22**	**2:27**	**2:28**	**2:34**
1:53	**2:07**	**2:10**	**2:17**	**2:23**	**2:26**	**2:32**	**2:36**	**2:39**	**2:42**	**2:47**	**2:48**	**2:54**

61B BRADDOCK-SWISSVALE

MONDAY THROUGH FRIDAY SERVICE

To Downtown Pittsburgh

Braddock Hills Shopping Center Propel School	Braddock 6th St at Braddock Ave	Braddock 7th St at Braddock Ave	Rankin Rankin Blvd at Hawkins Ave	Swissvale Monongahela Ave at Washington St	Regent Square Forbes Ave past S Braddock Ave	Squirrel Hill Forbes Ave past Murray Ave	Oakland Forbes Ave at S Craig St	Oakland Fifth Ave opp. Atwood St	Downtown Fifth Ave at Wood St
....	....	4:44	4:48	4:51	4:57	5:01	5:05	5:09	5:22
....	....	5:14	5:19	5:22	5:29	5:34	5:40	5:45	6:00
....	....	5:44	5:49	5:52	5:59	6:04	6:10	6:15	6:30
....	....	6:04	6:08	6:11	6:22	6:28	6:36	6:42	7:00
....	....	6:19	6:23	6:26	6:37	6:43	6:51	6:57	7:15
....	....	6:34	6:38	6:41	6:52	6:58	7:06	7:12	7:30
....	....	6:49	6:53	6:56	7:07	7:13	7:21	7:27	7:45
6:59	7:04	....	7:08	7:11	7:22	7:28	7:36	7:42	8:00
....	....	7:19	7:23	7:26	7:37	7:43	7:51	7:57	8:15
7:29	7:34	....	7:38	7:41	7:52	7:58	8:06	8:12	8:30
....	....	7:49	7:53	7:56	8:07	8:13	8:21	8:27	8:45
8:00	8:05	....	8:09	8:12	8:22	8:28	8:36	8:42	9:00
....	....	8:20	8:24	8:27	8:37	8:43	8:51	8:57	9:15
8:30	8:35	....	8:39	8:42	8:52	8:58	9:06	9:12	9:30
8:45	8:50	....	8:54	8:57	9:07	9:13	9:21	9:27	9:45
9:07	9:12	....	9:16	9:19	9:27	9:32	9:39	9:45	10:03
9:27	9:32	....	9:36	9:39	9:47	9:52	9:59	10:05	10:23
9:47	9:52	....	9:56	9:59	10:07	10:12	10:19	10:25	10:43
10:07	10:12	....	10:16	10:19	10:27	10:32	10:39	10:45	11:03
10:27	10:32	....	10:36	10:39	10:47	10:52	10:59	11:05	11:23
....	....	10:52	10:56	10:59	11:07	11:12	11:19	11:25	11:43
11:07	11:12	....	11:16	11:19	11:27	11:32	11:39	11:45	**12:03**
....	....	11:32	11:36	11:39	11:47	11:52	11:59	**12:05**	**12:23**
....	....	11:52	11:56	11:59	**12:07**	**12:12**	**12:19**	**12:25**	**12:43**

To Oakland - Squirrel Hill - Braddock

Downtown Fifth Ave at Wood St	Oakland Forbes Ave at Atwood St	Carnegie Museum Forbes Ave opp. S Craig St	Squirrel Hill Forbes Ave at Murray Ave	Regent Square Forbes Ave at S Braddock Ave	Swissvale Monongahela Ave opp. Washington St	Rankin Rankin Blvd opp. Hawkins Ave	Braddock Braddock Ave at 6th St	Braddock Hills Shopping Center Giant Eagle
5:22	5:33	5:36	5:41	5:46	5:52	5:55	5:59	6:05
6:00	6:11	6:14	6:19	6:24	6:30	6:33	6:37	6:43
6:30	6:41	6:44	6:49	6:54	7:00	7:03	7:07	7:13
7:00	7:16	7:19	7:25	7:30	7:37	7:40	7:44	7:50
7:15	7:31	7:34	7:40	7:45	7:52	7:55	7:59	8:05
7:30	7:46	7:49	7:55	8:00	8:07	8:10	8:14	8:20
7:45	8:01	8:04	8:10	8:15	8:22	8:25	8:29	8:35
8:00	8:16	8:19	8:25	8:30	8:37	8:40	8:44	....
8:15	8:31	8:34	8:40	8:45	8:52	8:55	8:59	9:05
8:30	8:46	8:49	8:55	9:00	9:07	9:10	9:14	....
8:45	9:01	9:04	9:10	9:15	9:22	9:25	9:29	9:35
9:00	9:14	9:17	9:24	9:30	9:37	9:40	9:44	....
9:15	9:29	9:32	9:39	9:45	9:52	9:55	9:59	10:05
9:30	9:44	9:47	9:54	10:00	10:07	10:10	10:14	....
9:45	9:59	10:02	10:09	10:15	10:22	10:25	10:29	....
10:03	10:17	10:20	10:27	10:33	10:40	10:43	10:47	....
10:23	10:37	10:40	10:47	10:53	11:00	11:03	11:07	....
10:43	10:57	11:00	11:07	11:13	11:20	11:23	11:27	....
11:03	11:17	11:20	11:27	11:33	11:40	11:43	11:47	....
11:23	11:37	11:40	11:47	11:53	**12:00**	**12:03**	**12:07**	....
11:43	11:57	**12:00**	**12:07**	**12:13**	**12:20**	**12:23**	**12:27**	**12:33**
12:03	**12:17**	**12:20**	**12:27**	**12:33**	**12:40**	**12:43**	**12:47**	**12:53**
12:23	**12:37**	**12:40**	**12:47**	**12:53**	**1:00**	**1:03**	**1:07**	....
12:43	**12:57**	**1:00**	**1:07**	**1:13**	**1:20**	**1:23**	**1:27**	**1:33**

61C MCKEESPORT-HOMESTEAD

MONDAY THROUGH FRIDAY SERVICE

To Downtown Pittsburgh

McKeesport Transportation Ctr Bay #2	**Duquesne** 2nd St at Grant Ave	**Kennywood Park** Kennywood Blvd opp. Hoffman Blvd	**Homestead** E Eighth Ave at Amity St	**Greenfield** Murray Ave past Hazelwood Ave	**Squirrel Hill** Forbes Ave past Murray Ave	**Oakland** Forbes Ave at S Craig St	**Oakland** Fifth Ave opp. Atwood St	**Downtown** Fifth Ave at Wood St
4:15	4:23	4:27	4:35	4:40	4:46	4:50	4:54	5:07
4:45	4:53	4:57	5:05	5:10	5:16	5:20	5:24	5:37
5:05	5:13	5:18	5:27	5:33	5:41	5:47	5:52	6:07
5:35	5:43	5:48	5:57	6:03	6:11	6:17	6:22	6:37
5:45	5:55	6:00	6:11	6:20	6:31	6:39	6:45	7:03
6:00	6:10	6:15	6:26	6:35	6:46	6:54	7:00	7:18
6:15	6:25	6:30	6:41	6:50	7:01	7:09	7:15	7:33
6:30	6:40	6:45	6:56	7:05	7:16	7:24	7:30	7:48
6:45	6:55	7:00	7:11	7:20	7:31	7:39	7:45	8:03
7:00	7:10	7:15	7:26	7:35	7:46	7:54	8:00	8:18
7:15	7:25	7:30	7:41	7:50	8:01	8:09	8:15	8:33
7:29	7:39	7:44	7:55	8:04	8:15	8:23	8:29	8:47
7:45	7:55	8:00	8:11	8:20	8:31	8:39	8:45	9:03
8:00	8:10	8:15	8:26	8:35	8:46	8:54	9:00	9:18
8:15	8:25	8:30	8:41	8:50	9:01	9:09	9:15	9:33
8:30	8:40	8:45	8:56	9:05	9:16	9:24	9:30	9:48
8:53	9:03	9:08	9:19	9:26	9:37	9:44	9:50	10:08
9:13	9:23	9:28	9:39	9:46	9:57	10:04	10:10	10:28
9:33	9:43	9:48	9:59	10:06	10:17	10:24	10:30	10:48
9:53	10:03	10:08	10:19	10:26	10:37	10:44	10:50	11:08
10:13	10:23	10:28	10:39	10:46	10:57	11:04	11:10	11:28
10:33	10:43	10:48	10:59	11:06	11:17	11:24	11:30	11:48
10:53	11:03	11:08	11:19	11:26	11:37	11:44	11:50	**12:08**
11:13	11:23	11:28	11:39	11:46	11:57	**12:04**	**12:10**	**12:28**
11:33	11:43	11:48	11:59	**12:06**	**12:17**	**12:24**	**12:30**	**12:48**

To Squirrel Hill - Homestead - McKeesport

Downtown Fifth Ave at Wood St	**Oakland** Forbes Ave at Atwood St	**Carnegie Museum** Forbes Ave opp. S Craig St	**Squirrel Hill** Forbes Ave at Murray Ave	**Greenfield** Murray Ave at Hazelwood Ave	**Homestead** E Eighth Ave at Amity St	**Kennywood Park** Kennywood Blvd at Hoffman Blvd	**Duquesne** 2nd St at Grant Ave	**McKeesport** Transportation Ctr Bay #2
5:07	5:18	5:21	5:26	5:33	5:38	5:47	5:51	5:58
5:37	5:48	5:51	5:56	6:03	6:08	6:17	6:21	6:28
6:07	6:18	6:21	6:26	6:33	6:38	6:47	6:51	6:58
6:37	6:48	6:51	6:56	7:03	7:08	7:17	7:21	7:28
7:03	7:19	7:22	7:28	7:37	7:43	7:52	7:56	8:04
7:18	7:34	7:37	7:43	7:52	7:58	8:07	8:11	8:19
7:33	7:49	7:52	7:58	8:07	8:13	8:22	8:26	8:34
7:48	8:04	8:07	8:13	8:22	8:28	8:37	8:41	8:49
8:03	8:19	8:22	8:28	8:37	8:43	8:52	8:56	9:04
8:18	8:34	8:37	8:43	8:52	8:58	9:07	9:11	9:19
8:33	8:49	8:52	8:58	9:07	9:13	9:22	9:26	9:34
8:47	9:03	9:06	9:12	9:21	9:27	9:36	9:40	9:48
9:03	9:17	9:20	9:27	9:36	9:42	9:52	9:56	10:04
9:18	9:32	9:35	9:42	9:51	9:57	10:07	10:11	10:19
9:33	9:47	9:50	9:57	10:06	10:12	10:22	10:26	10:34
9:48	10:02	10:05	10:12	10:21	10:27	10:37	10:41	10:49
10:08	10:22	10:25	10:32	10:41	10:47	10:57	11:01	11:09
10:28	10:42	10:45	10:52	11:01	11:07	11:17	11:21	11:29
10:48	11:02	11:05	11:12	11:21	11:27	11:37	11:41	11:49
11:08	11:22	11:25	11:32	11:41	11:47	11:57	**12:01**	**12:09**
11:28	11:42	11:45	11:52	**12:01**	**12:07**	**12:17**	**12:21**	**12:29**
11:48	**12:02**	**12:05**	**12:12**	**12:21**	**12:27**	**12:37**	**12:41**	**12:49**
12:08	**12:22**	**12:25**	**12:32**	**12:41**	**12:47**	**12:57**	**1:01**	**1:09**
12:28	**12:42**	**12:45**	**12:52**	**1:01**	**1:07**	**1:17**	**1:21**	**1:29**
12:48	**1:02**	**1:05**	**1:12**	**1:21**	**1:27**	**1:37**	**1:41**	**1:49**

61D MURRAY

MONDAY THROUGH FRIDAY SERVICE

To Downtown Pittsburgh

Waterfront Giant Eagle	**Waterfront** Waterfront Dr at Costco Drvwy	**Greenfield** Murray Ave past Hazelwood Ave	**Squirrel Hill** Forbes Ave past Murray Ave	**Oakland** Forbes Ave at S Craig St	**Oakland** Fifth Ave opp. Atwood St	**Downtown** Fifth Ave at Wood St
5:15	5:18	5:25	5:31	5:35	5:39	5:52
5:37	5:40	5:48	5:56	6:02	6:07	6:22
6:07	6:10	6:18	6:26	6:32	6:37	6:52
6:14	6:17	6:28	6:39	6:47	6:53	7:11
6:29	6:32	6:43	6:54	7:02	7:08	7:26
6:44	6:47	6:58	7:09	7:17	7:23	7:41
6:59	7:02	7:13	7:24	7:32	7:38	7:56
7:14	7:17	7:28	7:39	7:47	7:53	8:11
7:29	7:32	7:43	7:54	8:02	8:08	8:26
7:44	7:47	7:58	8:09	8:17	8:23	8:41
7:59	8:02	8:13	8:24	8:32	8:38	8:56
8:03	8:06	8:17	8:28	8:36	8:42	9:00
8:14	8:17	8:28	8:39	8:47	8:53	9:11
8:29	8:32	8:43	8:54	9:02	9:08	9:26
8:40	8:43	8:54	9:05	9:13	9:19	9:37
8:44	8:47	8:58	9:09	9:17	9:23	9:41
8:55	8:58	9:09	9:20	9:28	9:34	9:52
8:59	9:02	9:13	9:24	9:32	9:38	9:56
9:08	9:11	9:20	9:31	9:38	9:44	10:02
9:19	9:22	9:31	9:42	9:49	9:55	10:13
9:24	9:27	9:36	9:47	9:54	10:00	10:18
9:44	9:47	9:56	10:07	10:14	10:20	10:38
9:59	10:02	10:11	10:22	10:29	10:35	10:53
10:04	10:07	10:16	10:27	10:34	10:40	10:58
10:09	10:12	10:21	10:32	10:39	10:45	11:03
10:19	10:22	10:31	10:42	10:49	10:55	11:13
10:24	10:27	10:36	10:47	10:54	11:00	11:18
10:44	10:47	10:56	11:07	11:14	11:20	11:38
11:04	11:07	11:16	11:27	11:34	11:40	11:58

To Squirrel Hill - Waterfront

Downtown Fifth Ave at Wood St	**Oakland** Forbes Ave at Atwood St	**Carnegie Museum** Forbes Ave opp. S Craig St	**Squirrel Hill** Forbes Ave at Murray Ave	**Greenfield** Murray Ave at Hazelwood Ave	**West Homestead** Waterfront Dr opp. Costco driveway	**Waterfront** Giant Eagle	**Waterfront** Lowe's Home Center
5:52	6:03	6:06	6:11	6:18	6:23	6:27	6:29
6:22	6:33	6:36	6:41	6:48	6:53	6:57	6:59
6:52	7:03	7:06	7:11	7:18	7:23	7:27	7:29
7:11	7:27	7:30	7:36	7:45	7:51	7:55	7:57
7:26	7:42	7:45	7:51	8:00	8:06	8:10	8:12
7:41	7:57	8:00	8:06	8:15	8:21	8:25	8:27
7:56	8:12	8:15	8:21	8:30	8:36	8:40	8:42
8:11	8:27	8:30	8:36	8:45	8:51	8:55	8:57
8:26	8:42	8:45	8:51	9:00	9:06	9:10	9:12
8:41	8:57	9:00	9:06	9:15	9:21	9:25	9:27
8:56	9:12	9:15	9:21	9:30	9:36	9:40	9:42
....	....	....	....	....	....	....	....
9:11	9:25	9:28	9:35	9:44	9:50	9:54	9:56
9:26	9:40	9:43	9:50	9:59	10:05	10:09	10:11
....	....	....	....	....	....	....	....
9:41	9:55	9:58	10:05	10:14	10:20	10:24	10:26
....	....	....	....	....	....	....	....
9:56	10:10	10:13	10:20	10:29	10:35	10:39	10:41
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
10:18	10:32	10:35	10:42	10:51	10:57	11:01	11:03
10:38	10:52	10:55	11:02	11:11	11:17	11:21	11:23
....	....	....	....	....	....	....	....
10:58	11:12	11:15	11:22	11:31	11:37	11:41	11:43
....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....
11:18	11:32	11:35	11:42	11:51	11:57	**12:01**	**12:03**
11:38	11:52	11:55	**12:02**	**12:11**	**12:17**	**12:21**	**12:23**
11:58	**12:12**	**12:15**	**12:22**	**12:31**	**12:37**	**12:41**	**12:43**

Current Issues

One major issue that Pittsburgh Port Authority faces is about the operating cost per passenger and the revenue generated per passenger. Even though Port Authority is supposed to provide fixed-time bus services, the buses are almost guaranteed to be off-schedule due to unexpected traffic or weather conditions. The frequent off-schedule situation has caused inconvenience to students' daily life, resulting in a large number of students switching to Uber or Lyft as their choice of transportation. Under this scenario, Port Authority is losing its customers everyday and with the operating cost still remaining the same, the company is not generating as much profit as they used to anymore.

As we proceed with this project, we decide to analyze and optimize Port Authority's current cost/revenue situation. Thus, our research question is: How many buses should Port Authority dispatch on the road everyday in order to minimize operating cost and maximize revenue generated? For the simplicity of this project, we defined some assumptions and constraints as discussed in the "Assumptions" and "Constraints" sections.

Possible Solutions

We break our solutions down into two parts. The first part is "Base Case" and the second part is "Simulation Method". Within "Base Case", we explain in detail the simplest case under our research question. Graph, formula and real numbers are used here to demonstrate profit generated during different time periods. Within "Simulation Method", we simulate a much more complex and close to reality scenario. Based on predefined assumptions and constraints, we generate a real-time model where students are being continuously added and buses are continuously moving forward from one station to another. With the base case served as a foundation, we hope to use this simulation method to create a more realistic model and give readers a better idea on how to optimize the number of buses to maximize profit for Port Authority.

Base Case

Since the data set is too large and the variables are too many, we first use a base case to present a clear and simplified connections among variables. In the base case, we reduce the number of stops to only one stop and types of buses to only one type. To be more specific, in the base case we claim that this type of bus only picks up students at one stop. Furthermore, we make a few assumptions based on this claim:

1. The time interval that this type of bus arrives at the bus stop is a fixed **'T'** minutes, and we denote each time interval as T_1, T_2, T_3, …
2. Every time the bus comes, it picks up a number of students, leaving **'H'** students at the bus station. Since the stop is randomly chosen (i.e., not necessarily the first stop in the bus route), the number of students already on the bus variates, hence the number of empty seats on the bus variates. So **H** is a random non-negative variable.
3. The capacity of an empty bus is a fixed **'C'** (students).
4. We denote the students' increase rate at the bus stop as **'CR'**. **CR** is fixed.
5. Students are willing to wait a fixed **'L'** minutes for the bus before they get impatient and leave with Uber or Lyft instead of continue waiting at the bus stop.
6. The fee of each ride is a fixed **'d'** dollars.

The graph of the base case sketches as follows:

Number of Students at the Bus Stop

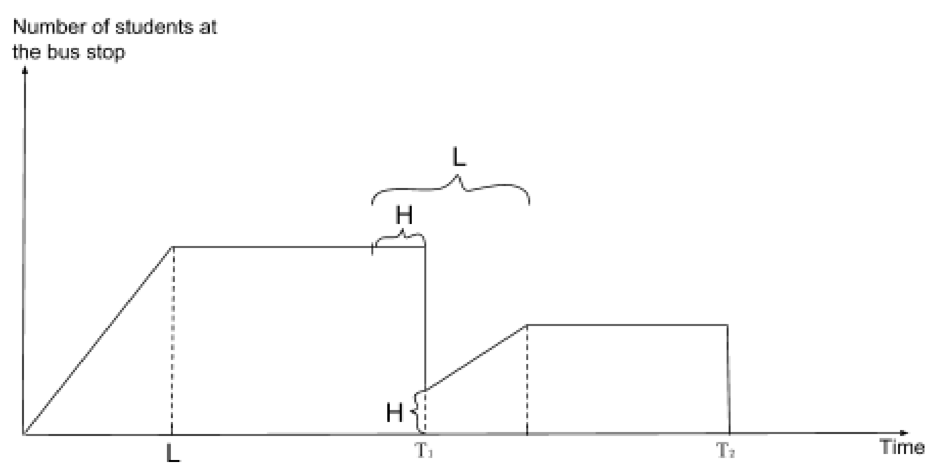


Figure 1: Base Case

For the sake of calculation, we assume the students' coming rate to the bust stop is 2 students/min, student's maximum willingness to wait is 10 minutes, the length of each time interval is 20 minutes, bus capacity is 15 students, fee for each ride is $1.

During the first 10 minutes, the student coming rate is 2 students/min. And after 10 minutes, students start to leave with Uber, so the student coming rate and leaving rate cancel off with each other. Hence the slope becomes 0. At the end of T_1=20 minutes, the bus comes and picks up 15 students, leaving 10*2-15=5 students at the bus station. Therefore, the student increase rate at the bus stop CR_2 at second interval is CR_1 from the previous period less the student leaving rate equals $CR_1 - H_1/(L - H_1)$. And the loop stops when H is nonpositive, meaning the students at the bus stop are all picked up by the bus.

During T_1,

$H_1 = CR * L - capacity = 2 * 10 - 15 = 5\ students$

$profit = min(capacity,\ CR_1 * L) *\ \$1 = min(15, 2 * 10) = \$15$

During T_2,

$CR_2 = CR_1 - H_1/(L - H_1) = 2 - 5/(10 - 5) = 1\ students/min$;

$H_2 = CR_2 * L - capacity = 1 * 10 - 15 =\ -5\ students$, meaning all the students can be picked up by the bus and there are still 5 seats empty ;

$profit = min(capacity,\ CR_2 * L) *\ \$1 = min(15, 1 * 10) = \$10$.

Now that we have explained the base case, and before we move onto the more complicated simulation method, the readers need to understand the assumptions and constraints that we based our simulation model on.

Assumptions

In order to make the problem more realistic and closer to the real life situation, but at the same time not too complicated for us to simulate, we make some assumptions to the problem. The first such assumption is about the route. Taking into account of the fact that the buses go through CMU like 61ABCD, we think of the bus routes of type A and B the same and the bus route of the type C and D the same. And we also take into account that buses ABCD all share the same routes in part of their itinerary. So our design of the route is as such: 15 stations in total, where all of ABCD buses cover the same first 10 stations and they go separate ways after the 10th station, where buses AB go to the same last 5 stations while CD go to the other 5 stations. And the route is circular, meaning that after each of the buses finish their 15-station route, they will complete their itinerary by going back to station one and then keep going. Also we make the capacity assumption for different type of buses. Unlike in the base case where we had no capacity roof for buses, we set the capacity of buses A and B at 40 students per bus. Similarly, the capacity of type C and D has a 55 students limit. It is also important to note that the bus service is only provided for students. In other words, only CMU students take these buses.

One of the most complicated assumptions we make for the problem is that the intervals between each stations are different while the buses all travel in the same speed. To break it down, this means that to travel from station 1 to station 2, buses ABCD will spend the same amount of time but the travel time between stations 1 and 2 is different from that between station 2 and 3. We randomly generate the intervals between the stations by time. So it could be like that the interval between station 1 and 2 is 3 minutes while the interval between station 3 and 4 is 6 minutes. We do realize that this is basically ignoring the speed limits and traffic, but if you think about the Pittsburgh traffic, it really is not that bad. This assumption also entailed that the time each bus spend at the stations can be ignored. We also assume that at the beginning, one single bus is "generated" randomly every 10 minutes from the start station.

The next assumption we make is about the student. We assume every minute, at each station, 3 more students show up to wait for the bus. This is probably a dialed up parameter in the sense that on average in real life, the student coming rate at each station should be less than 3. Also, there are tricky cases where one student may have to change a bus at some point in order to make the final destination. For example, they might get off of bus A at some point to wait for but C. And in this case, they abandon their original student ID we assigned to the student when they got on to bus A and they acquire a new student ID when they get on to bus C.

Constraints

The followings are the constraints given to our problem in order to do the simulation. We try to keep in mind the fact that this problem should be solvable and that the results should make sense as well as being economically feasible. The first constraint we introduce is that each student has a different destination which we will pick randomly. The reason why this is a constraint and not an assumption is that this limits the bus choice for a student. And another constraint is that the operation cost of each bus is 300 dollars per day, including the payment for the driver and the cost of fuel. Since we are only trying to figure out the number of buses that will maximize our revenue, we don't want to consider too many buses, say like 100 buses goes around in Pittsburgh every day is just too much for this city. So we consider up to 50 buses. And we consider a time span of 800 minutes for the purpose of our study. Last but not least, we say that one student will wait 10 minutes for a bus before he or she run out of patience and leave the station and take Uber or Lyft instead.

Simulation Method

To find the bus number that optimizes the total profit, we simulate the real-time situation by continuously adding students to each bus stop, moving each bus forward to the next stop, and removing students from the bus once they reach their destination. Here we will introduce the general code skeleton for our simulation method. In general, we have three helper classes, *student class*, *bus class* and *stations class*, in which we define the specific parameters and have some specific methods.

1.1 Bus Class:

Each bus object has parameters **`busId`**, **`busType`**, **`capacity`**, **`totalStudents`**, **`stationList`**, **`stationIdx`**, **`atStation`**, **`stationInterval`** dictionary and **`student`** dictionary. The **`busId`** is uniquely assigned to each bus, and the **`busType`** can only be one out of the all possible bus types (i.e. [A,B,C,D]). The capacity of bus is assigned corresponding to the **`busType`** (as we stated in the assumption part). The **`stationList`** defines all the stations

ID that the bus will visit in its routine and it is ordered by the station ID. For the **stationList**, each of the bus will have the same first ten common stations and then the remaining 5 stops may differ among different type of buses. For instance, for bus1 of type A we have **bus1.stationList = [0,1,2,3,4,5,6,7,8,9,10,11,12,13,14]** and for bus2 of type B we have **bus2.stationList = [0,1,2,3,4,5,6,7,8,9,15,16,17,18,19]**, where the last five elements in the lists are different. The **stationIdx** is the index that **stationList[stationIdx]** refers to the station ID we are currently at. If the parameter **atStation** is true if a bus in at one station. For instance, **bus2.stationIdx = 11**, **bus2.stationList[11] = 15**, and if **atStation = True**, then bus2 is currently at station with station ID 15, otherwise bus2 is in the process of moving from station with ID 15 to next station.And the student dictionary has key as the student ID and value as student object.

Because it's hard to compute the time interval between each station, we randomly generate fixed time interval between each stops. And the **stationIntervalDict** has the key as the stationed and value as the time taken to transport to next stop. As we assume all routines are circular, then for the last stop in the stationList of each bus, the value associated with it in the **stationIntervalDict** is the time to go back to initial station 0. Additionally, for each bus object, it has method **station()** to return its current station (or the previous station it just left) and method **isNotFullandAtStation** returns true when the bus has empty space and is currently not in the process of moving to next station. Also, each bus has **moveBus** method to move toward next station by decrementing the count in **stationIntervalDict** and it also has **pickUpStudents** method to add new student object to its student dictionary.

1.2 Student Class:

Each student object has parameters: **studentID**, **buses**, **timeArrival** and **numStopsToDestination**. As stated in assumption part, we assume the student ID for each student is unique. The buses parameter defines the **busType** that the student can actually take in an array. For instance, if a student can only take bus A and bus B, then the buses parameter will be [A,B]. The **numStopsToDestination** defines the number of stops the student need to stop by before they get off the bus. For instance, if **numStopsToDestination = 1**, then the student will get off the bus at the next station. For each student, the most important method is **needGetOff** function, which checks whether the student has reached his/her final destination. Since it's hard for us to collect the real-life data, we have a function **generateRandomStudent** to generate student object with **studentId** and **timeArrival** as input. And we randomly choose number between 1 and (total number of stops - 1) as **numStopstoDestination** and randomly select the **busType** the student can take.

1.3 Station Class:

Each station object has **stationID** and **student** dictionary as parameters. And for each station, we have **addStudent** method to add a student coming to the station into the corresponding **student** dictionary. Similarly, we have a **removeStudent** method to remove the given student object from the **student** dictionary.

1.4 Main Class:

Here is pseudocode of our main function, where we iteratively to try different number of buses using the simulation method:

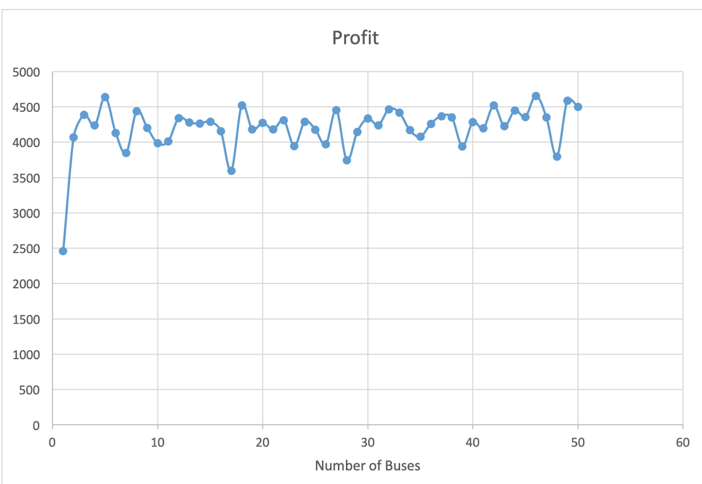
```
def main():
    construct station list
    initialize maximum profit = 0
    intialize maximum_profit_bus_number = -1
    for number in busNumbers:
        constuct a busList of all bus object
        set current time to 0
        loop until current time == total time:
            increment current time
            for each station in stationList:
                add new andonmly generated students to station
                for each bus in busList:
                    if bus is at this station and bus is not full:
                        pick up all students the bus can take
                        increment
                        remove those students from station
                    move bus toward nextStation

                for all students left at station:
                    if some student wait more than uberWaitingTime,
                    remove them from station
        calculate the profit, compare and update maximum profit
        update maximum_profit_bus_number
    return maximum profit, maximum_profit_bus_number
```

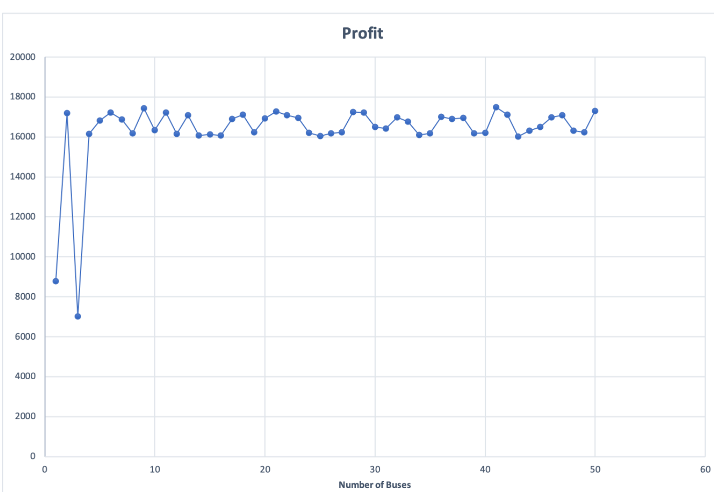
Result Analysis

As stated in our research question, we want to optimize the number of buses to use every day in order to maximize profit. Note that here profit refers to revenue generated from students paying to get on the bus minus the fixed operating cost of the bus per day. In the following part of this section, we present two result plots demonstrating how profit change according to the different number of buses used. As a reminder, our simulation is based on the following main assumptions:

(1) Total time span we study is 800 minutes
(2) Each student pay a fixed fee of $3 to get on the bus
(3) Each bus has a fixed operating cost of $300 per day

Now we look at the profit chart for profits against number of buses.





The above two plots are generated from running the simulation twice. Notice that the y-axis indicating the profit are quite different in range from the plot on the top to the one on the bottom. There are two reasons for this discrepancy in the amount of profit generated.

(1) Since the design of our model largely depends on randomly picking parameters, the results of one simulation run could be very different from another run of the simulation.
(2) Potential profit depends on the **`numStopToDestination`** parameter within the student class, i.e. how many stops a student still has until he/she reaches the final destination. In this case, when **`numStopToDestination`** is larger, this means that the bus would pick up fewer students along the way, thus leading to less profit.
(3) Potential profit also depends on the travel time interval between two adjacent stops. If the travel time interval is smaller, then buses move to the next station faster, the amount of students who switch to Uber or Lyft will be smaller, hence leading to more profit.

Other than the fact that the two plots display quite different y-axis ranges, we observe that the general trends of both plots are quite similar. Profit starts off low when only a small number of buses are used (less than 5 buses), but then profit starts to increase as we use more buses. There is a certain amount of fluctuations in the plots, but in general, we can see that the trends display an upward shift, indicating an increase in profit as we use more buses.

Looking at the two plots together, we conclude that the most amount of profit will be achieved by using 40-50 buses on the road per day.

Future Plans

Although we are able to determine the range of the number of buses to use to maximize profit, our whole model and simulation are based on heavy assumptions, constraints and simplification of the real-world problem. The real problem that Port Authority faces regarding operating cost and revenue takes into account real-time traffic situations. During rush hours, the buses are almost guaranteed to be late and that is why we often see huge amount of students waiting for the buses between 5pm and 6pm everyday.

A more realistic and well-defined model can be included into our future research plans by contacting Port Authority, getting real data on the amount of people who ride 61ABCD everyday, calculating the average time that buses move from one station to another and so on…

Overall, our solution demonstrates that the Pittsburgh Port Authority bus service can potentially be optimized to benefit both the customers and Port Authority itself. With the optimal number of buses on the road everyday, students' waiting time can be shortened and the maximum profit can be ensured for Port Authority.

References

[1] Sample bus schedule for 61A and 61B. Retrieved from
http://www.portauthority.org/rt/61a.pdf

[2] Sample bus schedule for 61C and 61D. Retrieved from
http://www.portauthority.org/rt/61c.pdf