**Inventory Optimization of Ready Meals at Entropy**

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**Introduction**

Entropy is a convenience store on Carnegie Mellon campus located in the University Center, and it has always been the go-to place for Carnegie Mellon students when they need snacks, beverages, basic medicines, and quick ready meals.

However, Entropy seems to have an inventory management problem with the ready meals. While many ready meals would sit for 3 to 5 days before some student purchase these meals, other more popular ready meals, such as chicken parmesan sandwich and California roll, sometimes become out of stock. As a group we decide to study the management of ready meals inventory at Entropy and devise an optimal strategy for the inventory management.

**Data Collection**

We tried to understand the operations of Entropy and get some data to work with before building our model, and we found some interesting facts. First of all, all the ready meals at Entropy are provided by CulinArt, a national company that offers dining services, and rather than hiring a store manager to manage the inventory at Entropy, CulinArt manages the inventory at a regional level. This fact seems to explain why there is such poor inventory management, since the regional manager definitely lacks knowledge of what has been popular or not popular at Carnegie Mellon. Secondly, ready meals are put up at Entropy twice a week, and we shall use this information later when we build the model. Due to the fact that CulinArt is a big national company and it hires regional manager to manage Entropy’s inventory, we had trouble getting data from CulinArt in a timely manner. Finally we decide to build the data ourselves after talking to staff at Entropy, so that we can estimate some data we need, including the weekly demand, ordering cost, inventory carrying charge, and the unit cost. We finally chose to look at 15 different ready meals with 5 being sandwich/salad products and the rest being sushi products, and the data we used could be found in Appendix A.

**Assumptions for Modeling**

There are essentially 4 key assumptions we make for this inventory optimization problem. First of all, we assume that the demand of students is constant over weeks. Secondly, we assume that the max number of orders Entropy can make per week is 2, based on the fact that ready meals at Entropy are put up twice a week. And this assumption further makes a half week the unit of time in our model. The third assumption we make is that there is limited space at Entropy to store ready meals in the inventory, and after talking to the staff we think it’s reasonable to assume that Entropy can store up to 150 sandwich/salad boxes in the inventory. And the final assumption we make is that a sandwich box takes up twice as much space as a sushi box.

**Modeling for Inventory Optimization**

We chose to start modeling with Wilson’s lot size equation, and the equation is as follows:

$$K=\frac{A}{T}+\frac{I\*Q}{2}$$

In the equation above, K stands for the average cost; A stands for the ordering cost; T stands for the time intervals between every 2 orders; I stands for the inventory carrying charge; Q stands for the ordering quantity for each other. We need to choose the optimal T and Q to minimize the cost K, and for our problem we take one step further to minimize cost for all of the 15 ready meals that we study. The new equation is as follows:

$$K\_{total}=\sum\_{i=1}^{i=15}\frac{A\_{i}}{T\_{i}}+\frac{I\_{i}\*Q\_{i}}{2}$$

There is one more constraint for us, which is the inventory space constraint. We denote the space that each kind of ready meals take as $f\_{i}$, and we set $f\_{i}$ for sandwich/salad products to be 1 and $f\_{i}$ for sushi products to be 0.5. Thus our total inventory space is 150, and the constraint could be written as follows:

$$\sum\_{i=1}^{i=15}f\_{i}\*Q\_{i}=F$$

$$F=150$$

We first try minimizing the cost using the Wilson’s equation, ignoring the inventory space constraint, and if the optimal solution does require more space than we have we will add the constraint to our modeling. And the equation for the optimal quantity to order is:

$$Q\_{i}=\sqrt{\frac{2\*A\_{i}\*λ\_{i}}{I\_{i}}}$$

After getting the optimal solution without the inventory space constraint, we calculated the space this solution would take and found that it exceeds what Entropy has (Appendix B). Thus, we try to implement the inventory space constraint using the Lagrange Multiplier Technique, which states that under the optimal solution there exists one θ that satisfies the equation below:

$$\frac{∂K}{∂Q\_{j}}=θ\frac{∂F}{∂Q\_{j}}$$

And on differentiation we see:

$$\frac{-λ\_{j}\*A\_{j}}{Q\_{j}^{2}}+\frac{I\_{j}}{2}=θ\*f\_{j}$$

Or $Q\_{j}=\sqrt{\frac{2\*λ\_{j}\*A\_{j}}{I\_{j}-2\*θ\*f\_{j}}}$

Now the value of $θ$ can be calculated by substituting the above equation into the inventory space constraint:

$$\sum\_{j=1}^{j=15}f\_{j}\*\sqrt{\frac{2\*λ\_{j}\*A\_{j}}{I\_{j}-2\*θ\*f\_{j}}}=150$$

And we solve such $θ$ using the solver equation and we get the new solution, which is shown in Appendix C. Since Q and T can only be integers for our problem, we further round the optimal Q\* down to the closest integer. We believe this ordering strategy would best capture the students’ demand for ready meals at entropy and also best utilize the limited inventory space.

**Stock-out Analysis**

As we state earlier in this paper, very often students couldn’t find their favorite ready meals on the shelf due to poor inventory management, and we want to study what’s the new stock-out probability under our optimal inventory strategy. Instead of assuming demand to be constant, now we assume that demand follows a normal distribution, and we use PERT formula to find the mean and the variance of the demand.

In the PERT formula, there are the best, the worst, and the most likely scenarios. We set the most likely scenario (denoted as m) to be the same as the data we use in the modeling, and we set the best scenario (denoted as b) to be 130% of that number and the worst scenario (denoted as a) to be 80% of that number. PERT calculates the mean and variance using the following formula:

$$Mean=\frac{a+4m+b}{6}$$

$$Variance=\frac{(b-a)^{2}}{36}$$

And after calculating the mean and variance, we use the NORMDIST function to find the stock-out probability, and the result is shown in Appendix D. We see that most items have relatively low stock-out probabilities, while chicken parmesan sandwich, California roll, and Boston roll have somewhat high stock-out probabilities. Based on this finding, we suggest Entropy monitor the inventory level of these 3 products more closely so that they can adjust to changing demand accordingly.

**Conclusion**

We suggest Entropy adopt our optimal ordering strategy, which could best cater the students’ demand for ready meals given the inventory space constraint. And according to our stock-out analysis, Entropy should watch the inventory level of chicken parmesan sandwich, California roll, and Boston roll and they can order more in a timely manner once these popular products go out of stock.

**Appendix A**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Item Name** | **λ: Demand (bi-weekly)** | **A: ordering cost** | **C: Unit cost per item** | **I: Inventory carrying charge** |
| Chicken parm sandwich | 17.5 | 5 | 3 | 0.2 |
| Ham sandwich | 7 | 5 | 3 | 0.2 |
| Roast beef sandwich | 10.5 | 5 | 3 | 0.2 |
| Salad box | 14 | 5 | 4 | 0.2 |
| Turkey sandwich | 10.5 | 5 | 4 | 0.2 |
| Sashimi 3 pc | 14 | 6 | 5 | 0.4 |
| Salmon roll | 10.5 | 6 | 5 | 0.4 |
| California roll | 17.5 | 6 | 5 | 0.4 |
| Avocado roll | 14 | 6 | 5 | 0.4 |
| Tuna roll | 10.5 | 6 | 5 | 0.4 |
| Spicy tuna roll | 14 | 6 | 6 | 0.4 |
| Cucumber roll | 7 | 6 | 6 | 0.4 |
| Edamame | 3.5 | 6 | 6 | 0.4 |
| Boston roll | 17.5 | 6 | 6 | 0.4 |
| Philly roll | 3.5 | 6 | 6 | 0.4 |

**Appendix B**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Item Name** | **λ** | **A** | **C** | **I** | **Q** | **f** | **Space taken** |
| Chicken parm sandwich | 17.5 | 5 | 3 | 0.2 | 29.58 | 1 | 29.58 |
| Ham sandwich | 7 | 5 | 3 | 0.2 | 18.71 | 1 | 18.71 |
| Roast beef sandwich | 10.5 | 5 | 3 | 0.2 | 22.91 | 1 | 22.91 |
| Salad box | 14 | 5 | 4 | 0.2 | 26.46 | 1 | 26.46 |
| Turkey sandwich | 10.5 | 5 | 4 | 0.2 | 22.91 | 1 | 22.91 |
| Sashimi 3 pc | 14 | 6 | 5 | 0.4 | 20.49 | 0.5 | 10.25 |
| Salmon roll | 10.5 | 6 | 5 | 0.4 | 17.75 | 0.5 | 8.87 |
| California roll | 17.5 | 6 | 5 | 0.4 | 22.91 | 0.5 | 11.46 |
| Avocado roll | 14 | 6 | 5 | 0.4 | 20.49 | 0.5 | 10.25 |
| Tuna roll | 10.5 | 6 | 5 | 0.4 | 17.75 | 0.5 | 8.87 |
| Spicy tuna roll | 14 | 6 | 6 | 0.4 | 20.49 | 0.5 | 10.25 |
| Cucumber roll | 7 | 6 | 6 | 0.4 | 14.49 | 0.5 | 7.25 |
| Edamame | 3.5 | 6 | 6 | 0.4 | 10.25 | 0.5 | 5.12 |
| Boston roll | 17.5 | 6 | 6 | 0.4 | 22.91 | 0.5 | 11.46 |
| Philly roll | 3.5 | 6 | 6 | 0.4 | 10.25 | 0.5 | 5.12 |
|  |  |  |  |  |  | **Total Space** | 209.47 |

**Appendix C**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Item Name** | **λ** | **A** | **C** | **I** | **f** | **Q\*** | **Space taken** | **T\*** | **Q\*\*** | **T\*\*** |
| Chicken parm sandwich | 17.5 | 5 | 3 | 0.2 | 1 | 18.36 | 18.36 | 1.05 | 18 | 1 |
| Ham sandwich | 7 | 5 | 3 | 0.2 | 1 | 11.61 | 11.61 | 1.66 | 11 | 1 |
| Roast beef sandwich | 10.5 | 5 | 3 | 0.2 | 1 | 14.22 | 14.22 | 1.35 | 14 | 1 |
| Salad box | 14 | 5 | 4 | 0.2 | 1 | 16.42 | 16.42 | 1.17 | 16 | 1 |
| Turkey sandwich | 10.5 | 5 | 4 | 0.2 | 1 | 14.22 | 14.22 | 1.35 | 14 | 1 |
| Sashimi 3 pc | 14 | 6 | 5 | 0.4 | 0.5 | 17.33 | 8.66 | 1.24 | 17 | 1 |
| Salmon roll | 10.5 | 6 | 5 | 0.4 | 0.5 | 15.01 | 7.50 | 1.43 | 15 | 1 |
| California roll | 17.5 | 6 | 5 | 0.4 | 0.5 | 19.37 | 9.69 | 1.11 | 19 | 1 |
| Avocado roll | 14 | 6 | 5 | 0.4 | 0.5 | 17.33 | 8.66 | 1.24 | 17 | 1 |
| Tuna roll | 10.5 | 6 | 5 | 0.4 | 0.5 | 15.01 | 7.50 | 1.43 | 15 | 1 |
| Spicy tuna roll | 14 | 6 | 6 | 0.4 | 0.5 | 17.33 | 8.66 | 1.24 | 17 | 1 |
| Cucumber roll | 7 | 6 | 6 | 0.4 | 0.5 | 12.25 | 6.13 | 1.75 | 12 | 1 |
| Edamame | 3.5 | 6 | 6 | 0.4 | 0.5 | 8.66 | 4.33 | 2.48 | 8 | 2 |
| Boston roll | 17.5 | 6 | 6 | 0.4 | 0.5 | 19.37 | 9.69 | 1.11 | 19 | 1 |
| Philly roll | 3.5 | 6 | 6 | 0.4 | 0.5 | 8.66 | 4.33 | 2.48 | 8 | 2 |
|  |  |  |  |  |  |  |  |  |  |  |
| **Theta** | -0.16 |  |  |  |  |  |  |  |  |  |

**Appendix D**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Item Name** | **a** | **m** | **b** | **mean** | **var** | **sd** | **Inventory level** | **Stock-out probability** |
| Chicken parm sandwich | 14 | 17.5 | 22.75 | 17.79 | 2.13 | 1.46 | 18 | 0.443 |
| Ham sandwich | 5.6 | 7 | 9.1 | 7.12 | 0.34 | 0.58 | 11 | 0.000 |
| Roast beef sandwich | 8.4 | 10.5 | 13.65 | 10.68 | 0.77 | 0.88 | 14 | 0.000 |
| Salad box | 11.2 | 14 | 18.2 | 14.23 | 1.36 | 1.17 | 16 | 0.065 |
| Turkey sandwich | 8.4 | 10.5 | 13.65 | 10.68 | 0.77 | 0.88 | 14 | 0.000 |
| Sashimi 3 pc | 11.2 | 14 | 18.2 | 14.23 | 1.36 | 1.17 | 17 | 0.009 |
| Salmon roll | 8.4 | 10.5 | 13.65 | 10.68 | 0.77 | 0.88 | 15 | 0.000 |
| California roll | 14 | 17.5 | 22.75 | 17.79 | 2.13 | 1.46 | 19 | 0.204 |
| Avocado roll | 11.2 | 14 | 18.2 | 14.23 | 1.36 | 1.17 | 17 | 0.009 |
| Tuna roll | 8.4 | 10.5 | 13.65 | 10.68 | 0.77 | 0.88 | 15 | 0.000 |
| Spicy tuna roll | 11.2 | 14 | 18.2 | 14.23 | 1.36 | 1.17 | 17 | 0.009 |
| Cucumber roll | 5.6 | 7 | 9.1 | 7.12 | 0.34 | 0.58 | 12 | 0.000 |
| Edamame | 2.8 | 3.5 | 4.55 | 3.56 | 0.09 | 0.29 | 4 | 0.065 |
| Boston roll | 14 | 17.5 | 22.75 | 17.79 | 2.13 | 1.46 | 19 | 0.204 |
| Philly roll | 2.8 | 3.5 | 4.55 | 3.56 | 0.09 | 0.29 | 4 | 0.065 |