Q1 Can the following shortest path problem be solved by the Dijkstra algo-
rithm? The edges of a digraph are colored Red and Blue. Suppose edge
lengths are non-negative, but a path can have at most $k$ red edges. Give
an explicit definition of path length.

Q2 Convert the following 3-dimensional assignment problem into a 2-dimensional
problem. There are objects $A = \{a_1, a_2, \ldots, a_n\}$, $B = \{b_1, b_2, \ldots, b_n\}$, $C =
\{c_1, c_2, \ldots, c_n\}$ and $A \cup B \cup C$ must be partitioned into $n$ triples with
one element from each of $A, B, C$ in each triple. The value of a triple
$\{a_i, b_j, c_k\}$ is given by $c_{i,j,k} = u_{i,j} + v_{i,k}$. The goal is partition the triples
at minimum total cost.

Hint: a partition into triples can be determined by two permutations
$\phi, \psi$ of $[n]$. In which case we have triples $(a_{i}, b_{\phi(i)}, c_{\psi(i)})$ for $i \in [n]$.

Q3 Suppose we color the elements of a set $A$ with $q$ colors. Let a subset of
$S$ be rainbow colored if all of its elements have a different color. Show
that the collection of rainbow colored sets forms a matroid.