Inventory Control

Model 1
Demand for a product in X units per period.

The fixed (administrative) cost of making an order is A.

The cost of keeping one unit of stock for one period is I.

Problem: minimize order + inventory cost.
What value of $Q, T$ minimizes total cost.

Average cost per period:

$$K = \frac{A}{T} + \frac{I_0}{2} = \frac{A_T}{Q} + \frac{I_0}{2}$$

Order cost + inventory cost

Convex

$$\frac{dk}{dQ} = -\frac{A_T}{Q^2}$$

Optimal $Q = \sqrt{\frac{2A_T}{I_0}}$ Wilson-Lat-Size Formula

Optimal $T = \sqrt{\frac{2A_T}{I_0}}$
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Model 2

Demand for a product is \( h \) units per period.

The fixed (administrative) cost of making an order is \( A \).

The cost of keeping one unit of stock for one period is \( I \).

Allowed to go out of stock and back order items. Pay penalty per period for an item out of stock.
What value of Q, T minimizes total cost.

Average cost per period

\[ K = \frac{A}{T} + \frac{T}{\frac{Q-S}{T}} + \frac{T}{T - 2} \cdot 5T = \frac{Q}{Q} + \frac{(\sqrt{2} - 0.5)I}{20} + \frac{5\pi}{20} + H(Q, S) \]

Order cost + maintenance + backorder cost \( \frac{Q}{T} \cdot \frac{Q-S}{T} \cdot \frac{1}{2} = 0.5 \text{ for solve.} \)
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Model 3

Demand for a product in $\lambda$ units per period.

The fixed (administrative) cost of making an order is $H$.

The cost of keeping one unit of stock for one period is $I$.

Problem: minimize order + inventory cost.
\[ K = \frac{A}{T} + \frac{Q}{2} = A \cdot \frac{1}{Q} \cdot \frac{(\text{Ordering cost} \times \text{Inventory cost})}{2} \]

\[ Q = \frac{HT}{2} \]

\[ T + T_2 = T \]

\[ (1-x)T_1 + h \]

\[ T_2 = h \]

Optimal \[ Q = \frac{H(T - h)}{2} \]

\[ K = K_0 \left( \frac{Q}{Q^*} \right)^2 \]