

Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 18.

Describe a Dynamic programming solution to the following problems:

- Q1** Suppose that in simple production problem, we do not have to meet demand immediately. Instead there is a penalty of b dollars per unit per period of demand not met. Suppose that we are only allowed to have at most B units of demand unfulfilled at any stage.
- Q2** An $m \times n$ rectangle of wood is to be cut into smaller rectangles. An $a \times b$ rectangle is worth $m_{a,b}$. The machine that cuts rectangles can only cut full length or full width. I.e. if after cutting there is an $x \times y$ rectangle then the machine can cut it into two rectangles $z \times y$ and $(x - z) \times y$ for some z or into two rectangles $x \times z$ and $x \times (y - z)$. All rectangles cut must have integral side lengths.
- Q3** Consider a 2-D map with a horizontal river passing through its center. There are n cities on the southern bank with x -coordinates $a(1) \dots a(n)$ and n cities on the northern bank with x -coordinates $b(1) \dots b(n)$. You want to connect as many north-south pairs of cities as possible with bridges such that no two bridges cross. When connecting cities, you can only connect city i on the northern bank to city i on the southern bank. Construct a Dynamic Programming solution to this problem. (You can assume that $a(1) < a(2) < \dots < a(n)$, but you **cannot** assume that $b(1) < b(2) < \dots < b(n)$. If both sequences are increasing, then the problem is trivial).
- Q4** You have to drive across country along a road of length L . There are gas stations at points $P_1, P_2, \dots, P_r$ along the route. Your car can hold g gallons of gasoline. At gas station i , the price of gas is p_i per gallon. If you drive at s miles per hour then you use up $f(s)$ gallons of gas per mile. You can assume that you have to drive at constant speed between stops. You start with a full tank of gas and you have an amount A to spend on the trip. Can you finish the trip in time at most T ?