

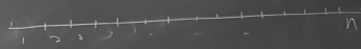
Production Problem: Machine Replacement

Demand in period  $t$  is  $d_t$

Maximum amount of stock =  $H$

Cost of making  $x$  units on a machine of age  $t$  is  $C(x, t)$

A new machine costs  $A$  to buy.



$$f(\text{???}) = \text{cost of decision} + \text{one period cost} + f'(\text{---})$$

↑  
cost of finishing to end.

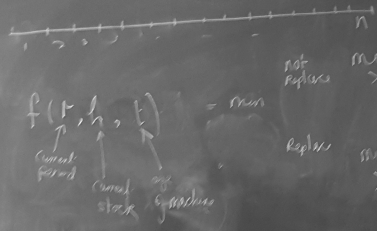
Production Problem; machine Replacement

Demand in period  $t$  is  $d_t$

Maximum level of stock =  $H$

Cost of making  $x$  units on a machine of age  $t$  is  $c(x, t)$

A new machine costs  $A$  to buy



not replace

$$\min_x [c(x, t) + f(r+1, h+x-d_t, t+1)]$$

replace

$$\min_x [A + c(x, 0) + f(r+1, h+x-d_t, t+1)]$$

### Breaking up a stick



Suppose value of stick  $[i, j] = \frac{v(i, j)}{O(1)}$ ,  $i < j$

Problem: chop into pieces at  $i_1, i_2, \dots, i_k$  to maximize

$$v(i_1, i_2) + v(i_2, i_3) + \dots + v(i_{k-1}, L)$$

$$f(L) = \max_{\text{Right-most cut } l} [v(l, L) + f(l)]$$

Solves problem in  $O(L^2)$  time.

Compute  $f(i_1, i_2), \dots, f(L)$

Suppose now, I need to cut stick into exactly  $k$  pieces



$$f(i, l) = \max_j [v(i, j) + f(j, l)]$$

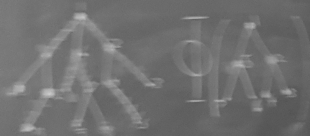
$i = 0, 1, \dots, L$

$l = i_1, i_2, \dots, k$

$= \#$  of cuts needed

Requires  $O(kL^2)$  computation.

Let  $G$  be a tree with root  $r$  and  $n$  nodes.



$$\sum_{k=1}^n \binom{n-1}{k-1} = 2^{n-1}$$

Example:  $f(n) = 2^{n-1}$

Suppose  $\sum_{k=1}^n \binom{n-1}{k-1} = 2^{n-1}$