

Integer Linear Programming

Many goals to linear programs
 also we require some of the
 variables to be integers.

eg. maximize $c^T x$
 subject to $Ax = b$
 $x \geq 0$
 $x_i \text{ integer } i=1, \dots, n$

Resource Allocation Problem

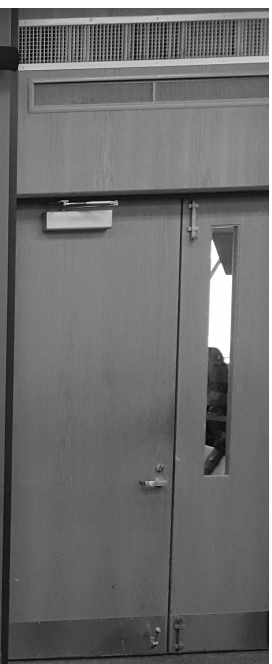
M time periods
 N possible projects that could be undertaken.
 Project j will earn c_j if undertaken.
 It will an expenditure of a_{ij} in period i , if undertaken.
 We have b_i available in period i to spend.
 Problem: Choose a set of projects to maximize revenue.

Let $x_j = \begin{cases} 0 & \text{not do project } j \\ 1 & \text{do project } j \end{cases}$

Maximize Revenue = $c_1 x_1 + c_2 x_2 + \dots + c_n x_n$ $\leftarrow c_j = \text{revenue from project } j$

Subject to expenditure = $a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \leq b_1, \dots, a_{m1} x_1 + \dots + a_{mn} x_n \leq b_m, \dots, n$

$0 \leq x_j \leq 1 \quad j=1, \dots, n$
 $x_j \text{ integer}$

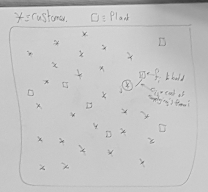


Simple Plant Location Problem

Company has to distribute goods to
 N customers

You want to build "Plant" to supply these customers.

You have identified m places where you could build your plant.

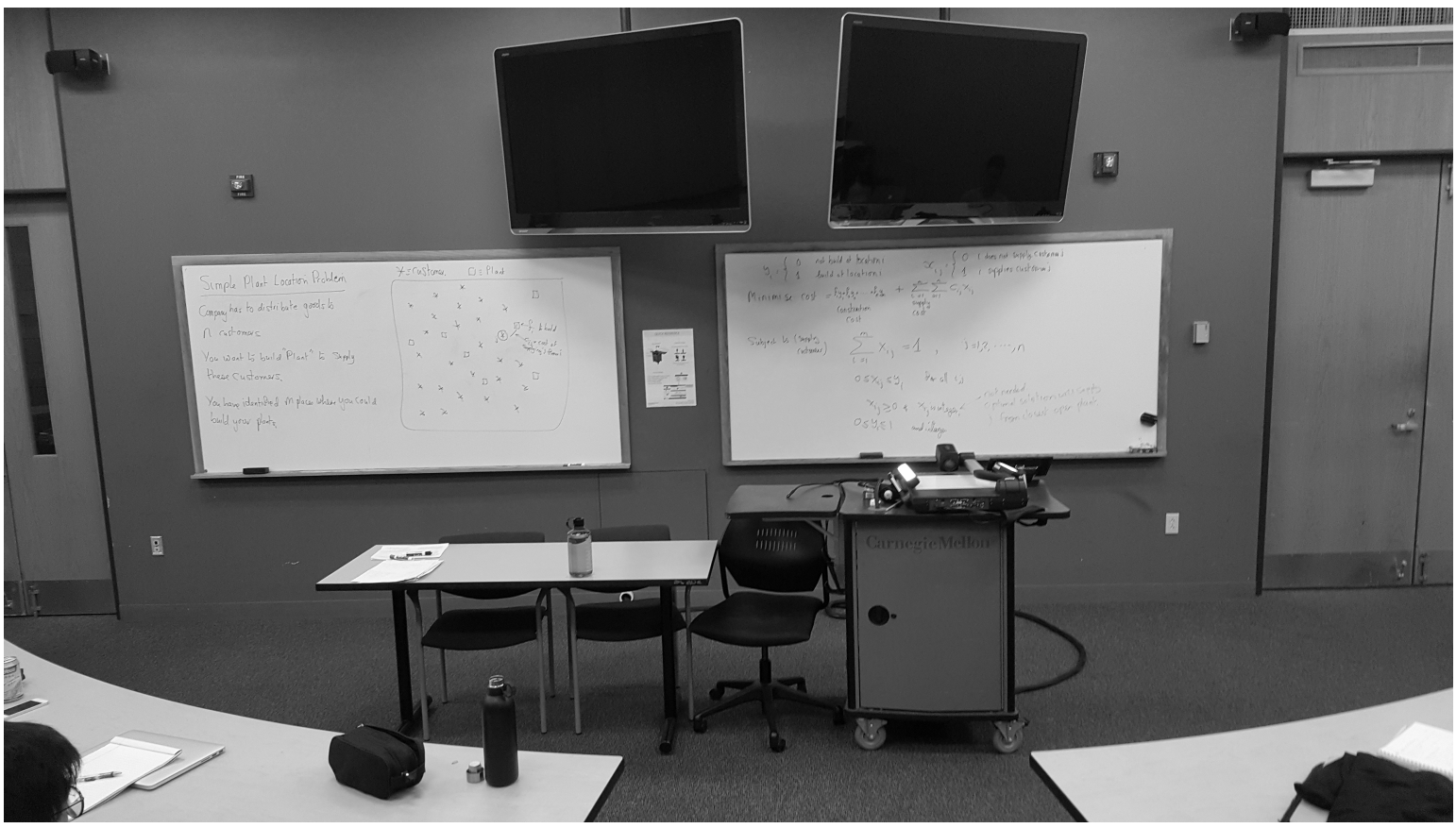


$y_i = \begin{cases} 0 & \text{not build at location } i \\ 1 & \text{build at location } i \end{cases}$
 $x_{ij} = \begin{cases} 0 & \text{does not supply customer } j \\ > 0 & \text{supplies customer } j \end{cases}$

Minimum cost = $\sum_{i=1}^m f_i y_i + \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$
fixed location cost variable cost

Subject to (supply customer)
 $\sum_{i=1}^m x_{ij} = d_j, \quad j=1, 2, \dots, n$
 $0 \leq x_{ij} \leq d_j, \quad \forall i, j$
 $0 \leq y_i \leq 1, \quad \forall i$

*not needed - optimal solution will satisfy
 - plant location will supply
 - from lowest cost plant*



Set Cover Problem

$$S_1, S_2, \dots, S_m \subseteq S$$

Each S_i has a cost c_i associated with it.

A cover $I \subseteq [m] = \{1, 2, \dots, m\}$

Such that

$$\bigcup_{i \in I} S_i = S$$

Problem: Compute minimum cost cover

$$c(I) = \sum_{i \in I} c_i$$

Arise in scheduling airline crews.

$$S = \{\text{flights operated in a period}\}$$

S_i : a set of flights that one crew can do.

