Infinite Horizon Model

So far we have assumed a fixed number of periods etc.

Production Model:

Costs

8	5	6	8	5	6	...	Strategy 1 — $6\frac{1}{3}$ per period
7	4	7	7	4	8	...	" 2 — better ← 6 per period; average cost per period
6	5	7	6	5	7	...	3 — average cost 6 per period

Inflation $\alpha < 1$

$7 + 4\alpha + 7\alpha^2 + 7\alpha^3 + \dots$ (2)

$6 + 5\alpha + 7\alpha^2 + \dots$ (3)

Model

A system has n states

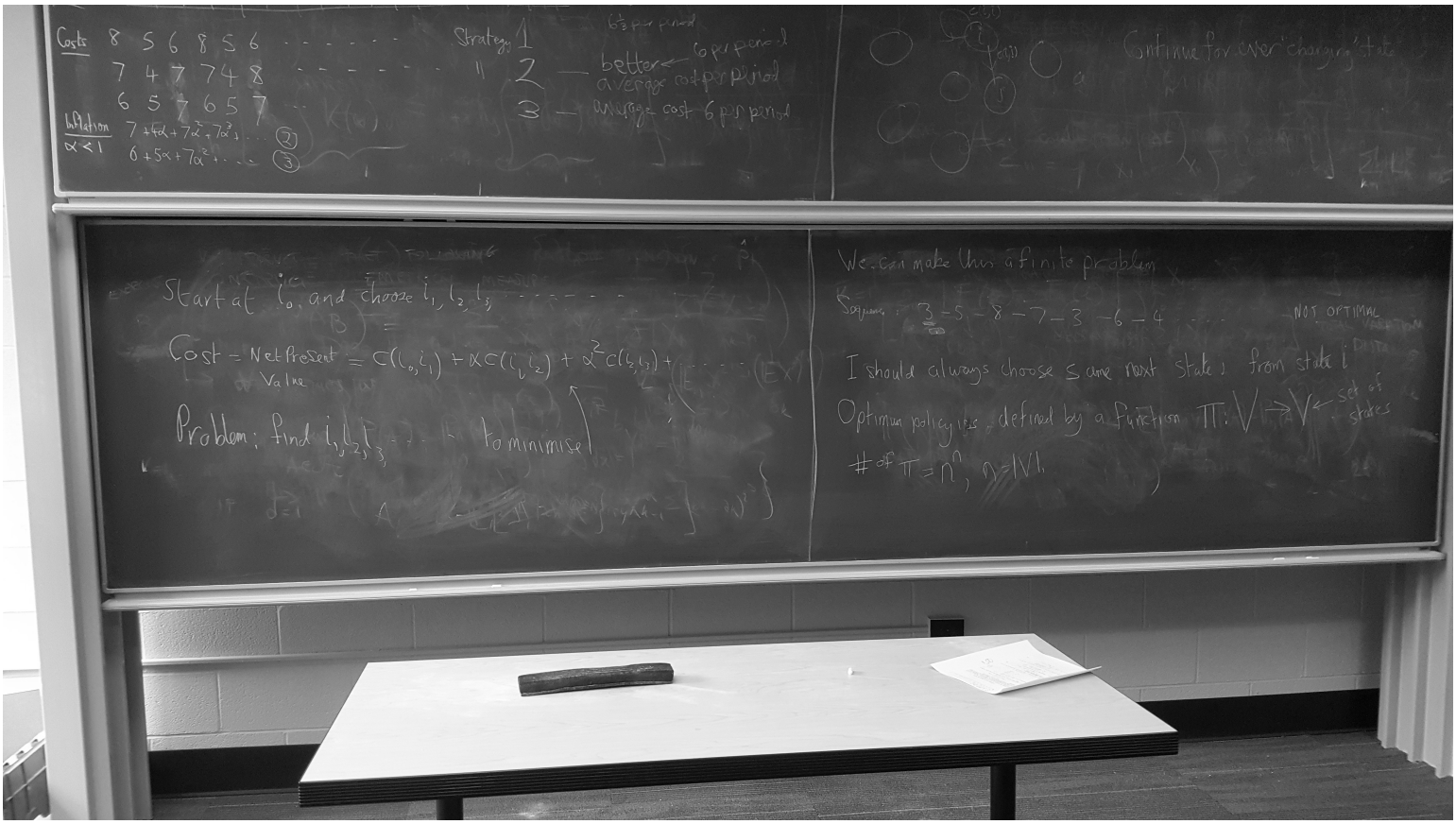
When in a state decide which is next state

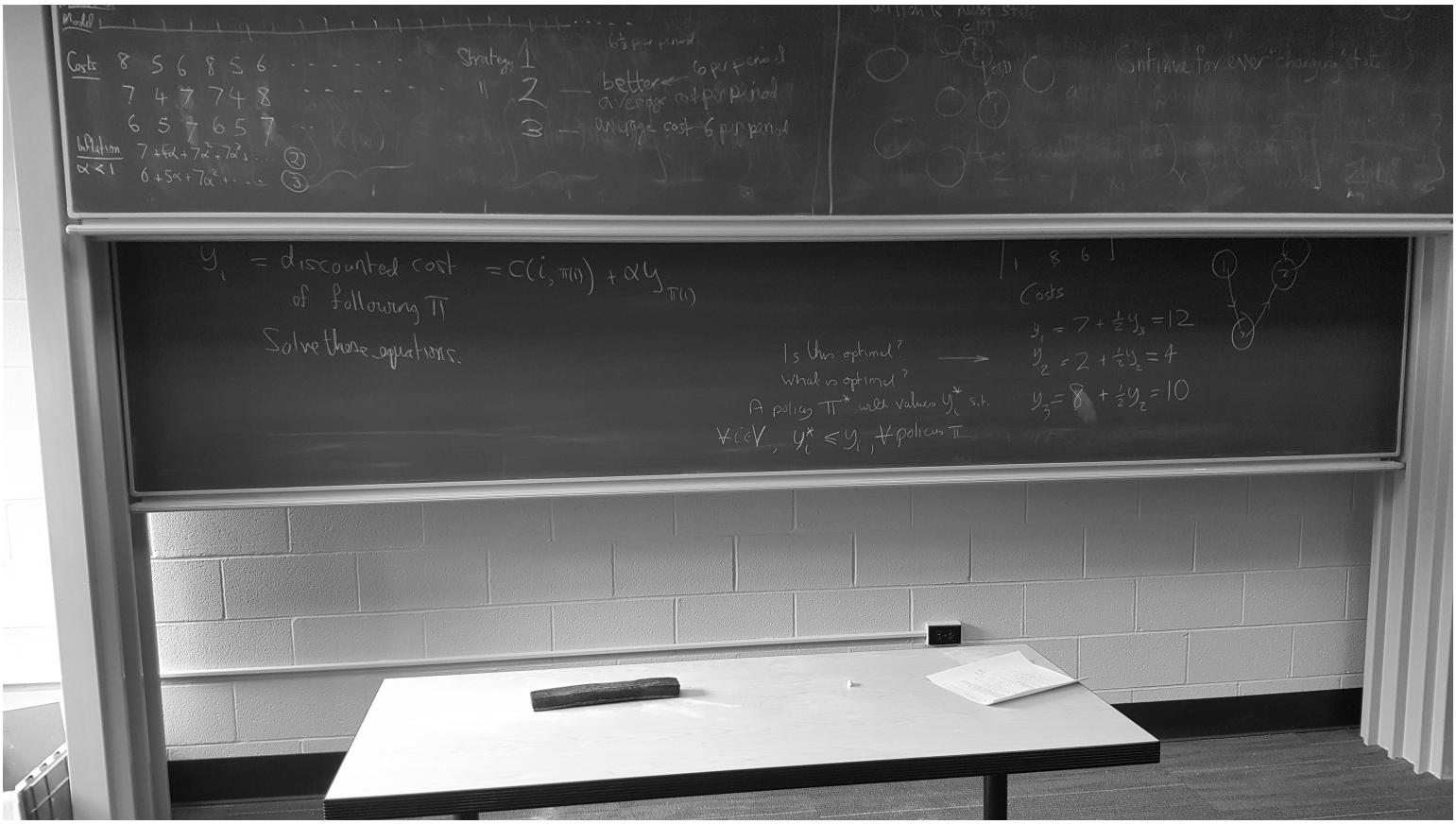
$c(i,i)$, $c(i,j)$

i = inventory level at start of a period

$i \xrightarrow{\text{Cost}(j-i+d)} j$

Continue for ever "changing" state


Costs 8 5 6 8 5 6 . . . . . Strategy 1
7 4 7 7 4 8 2
6 5 7 6 5 7 . . 3
Inflation 7 + 4α + 7α² + 7α³ + . . . ①
α < 1 6 + 5α + 7α² + . . . ③
better ← 6 per period
average cost per period
average cost 6 per period
Continue forever "changing" state
Start at i₀ and choose i₁, i₂, i₃ . . . . . .
Cost = Net Present Value = C(i₀, i₁) + αC(i₁, i₂) + α²C(i₂, i₃) + ... (EX)
Problem: find i₁, i₂, i₃ . . . to minimise
We can make this a finite problem
Sequence: 3 – 5 – 8 – 7 – 3 – 6 – 4 . . . . NOT OPTIMAL
I should always choose ≤ one next state j from state i
Optimum policy is defined by a function Π: V ⟹ V ← set of states
# of Π = n^n, n = |V|.


Costs 8 5 6 8 5 6 . . . . . Strategy 1
7 4 7 7 4 8 . . . . . 2 — better ← 6 per period
6 5 7 6 5 7 . . . 3 — average cost per period
Inflation 7 + 6α + 7α² + 7α³ + . . . (2)
α < 1 6 + 5α + 7α² + . . . (3)
Continue for ever "changing" state
y_i = discounted cost of following π $= c(i, \pi(i)) + \alpha y_{\pi(i)}$
Solve these equations.
Is this optimal?
What is optimal?
A policy π^* with values y_i^* s.t.
$\forall i \in V$, $y_i^* \leq y_i$, $\forall$ policies π
Costs
$y_1 = 7 + \frac{1}{2} y_3 = 12$
$y_2 = 2 + \frac{1}{2} y_2 = 4$
$y_3 = 8 + \frac{1}{2} y_2 = 10$