Production problem with random demands

Cost is $c(x)$ as before. H = maximum inventory

Demand in a period is uncertain. Penalty π for unmet demand

$\Pr[\text{demand} = d] = p_d$, in any period

Goal: minimise expected cost.

$f(r, h)$ = min. expected cost from periods $r, r+1, \dots, n$

$$= \min_x \left\{ c(x) + \sum_d p_d \left(\pi(d-h-x)^+ + f(r+1, (h+x-d)^+) \right) \right\}$$

decide on produc… we see deman…

$$f(r,h) = \sum_d p_d \min_x \left(c(x) + \pi(d-h-x)^+ + f(r+1, (h+x-d)^+) \right)$$

decide on prod… after seeing…

Goal: minimise expected cost.

Machine Replacement

Suppose p_t = probability that a machine of age t fails.

Assume $p_0 = 0$

$$f(r,t,h) = \min \begin{cases} \min\limits_{x} \Big(p_t \big(A + c(x,0) + f(r+1, 1, h+x-d_r)\big) + (1-p_t)\big(c(x,t) + f(r+1, t+1, h+x-d_r)\big)\Big) & \text{keep} \\ \min\limits_{x} \big[A + c(x,0) + f(r+1, 1, h+x-d_r)\big] & \text{replace} \end{cases}$$

$Pr[\text{demand} = d] = p_d$, in any period

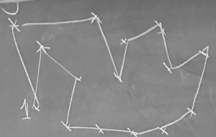
Goal: minimise expected cost.

$$f(r,h) = \sum_d p_d \min_x \left(c(x) + \pi(d-h-x)^+ + f(r+1, (h+x-d)^+) \right)$$

decide on production after seeing demand

Traveling Salesperson Problem (TSP)

n cities



c_{ij} = cost $i \longrightarrow j$

A tour T is a sequence

$1 = i_1, i_2, i_3, \ldots, i_n$

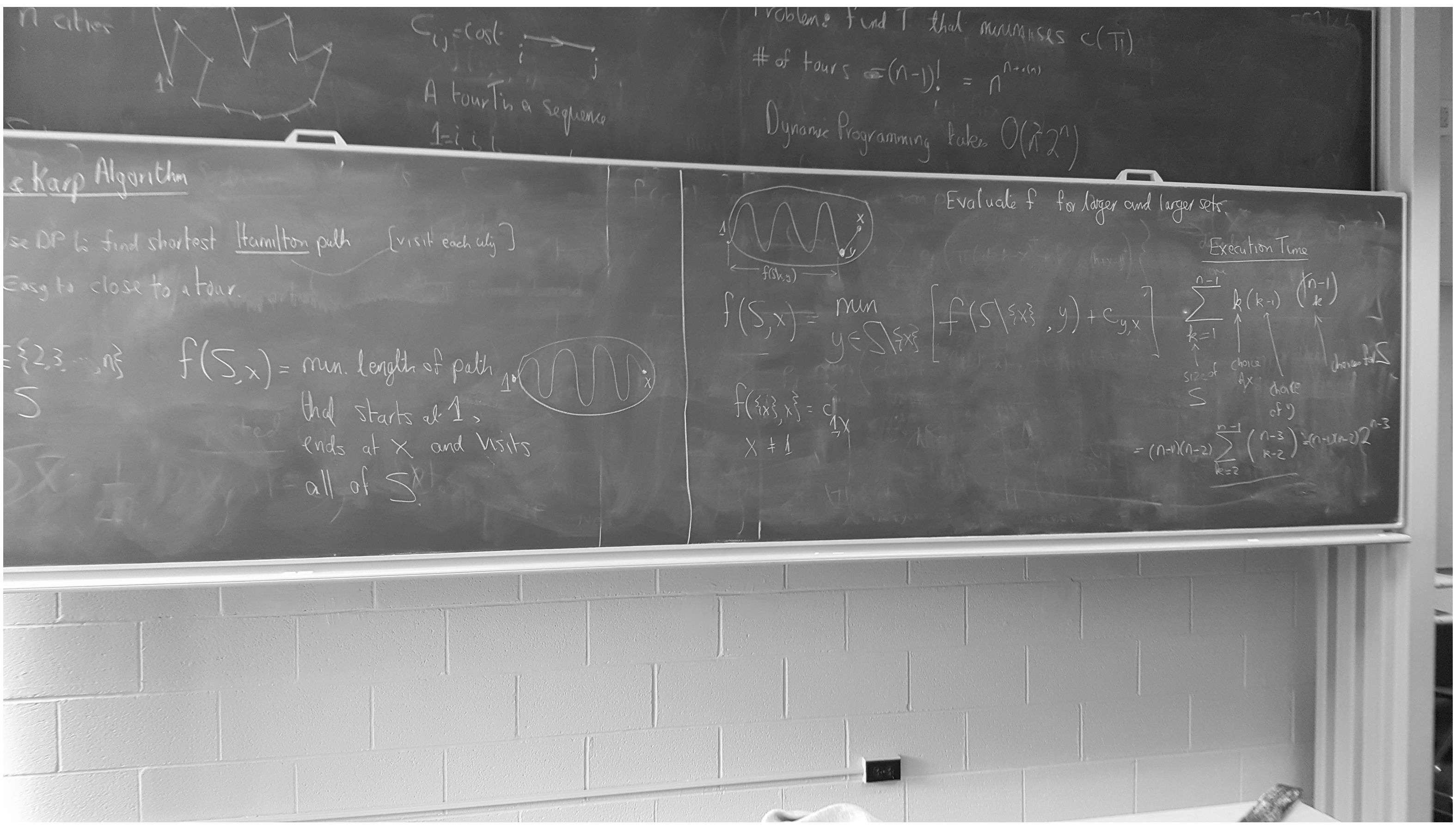
Salesperson must make a tour and visit each city exactly once

Cost $c(T) = c_{i_1 i_2} + c_{i_2 i_3} + \ldots + c_{i_{n-1} i_n} + c_{i_n i_1}$

Problem: find T that minimises $c(T)$.

# of tours $= (n-1)! = n^{n+o(n)}$

Dynamic Programming takes $O(n^2 2^n)$



n cities
c_{ij} = cost
A tour T is a sequence
Problem: find T that minimises c(T)
# of tours = (n-1)!
Dynamic Programming takes $O(n^2 2^n)$
Karp Algorithm
Use DP to find shortest Hamilton path [visit each city]
Easy to close to a tour.
f(S, x) = min. length of path that starts at 1, ends at x and visits all of S.
Evaluate f for larger and larger sets.
$f(S,x) = \min_{y \in S \setminus \{x\}} \left[f(S \setminus \{x\}, y) + c_{y,x} \right]$
$f(\{x\}, x) = c_{1,x}$
$x \neq 1$
Execution Time
$\sum_{k=1}^{n-1} k(k-1) \binom{n-1}{k}$
size of S
choice of x
choice of y
choices for S
$= (n-1)(n-2) \sum_{k=2}^{n-1} \binom{n-3}{k-2} = (n-1)(n-2) 2^{n-3}$