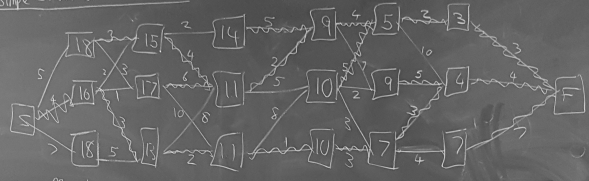


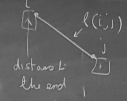
Dynamic Programming

Start with a problem P  
we construct "smaller" problems  
 $P_1, P_2, \dots, P_k$  and use solutions  
to solve P

Single Shortest Path Problem



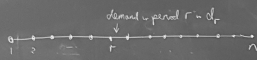
#paths =  $3^m$   
Our method  $m \times 9$



### Production Problem

Factory manufacturing a single item e.g. washing machines

Plan production over next  $n$  periods  
We want to minimise cost and meet demand.



Cost of making  $x$  units is  $C(x)$ .  
Assume you can store up to  $H$  units of inventory.  
Storing  $y$  units for a period costs  $ay$  for some  $a > 0$ .

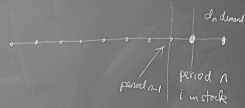
PH 22&c

Carnegie

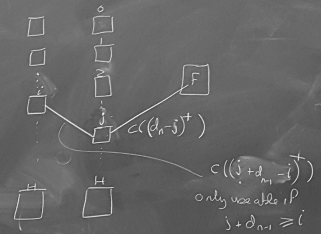


We reduce  $n$  period problem  $P$   
to smaller (fewer periods) problem  
and use answers to solve  $P$ .

We reduce stock problem to  
shortest path problem



Cost is  $C(\max\{0, d_n - i\})$   
Assume  $c(0) = 0$



Plan production over next  $n$  periods

We want to minimise cost  
and meet demand.

Cost of making  $x$  units is  $c(x)$

Assume you can store up to  $H$   
units of inventory.

Storing  $y$  units for a period costs  
for some  $a > 0$ .

To solve a period problem we will need to solve  $n-1$  period problems.  $t=0, 1, \dots, n-1$  for all possible values starting inventory

We want to develop a recurrence for  $f(r, i)$  = min. cost of meeting demand  $d_1, d_2, \dots, d_n$  starting with  $i$  in stock

$$f(r, i) = \min_x \left\{ \underbrace{c(x)}_{\text{production cost}} + \underbrace{h_1(i-x-d)}_{\text{inventory cost}} + \underbrace{f(r+1, i+x-d_r)}_{\text{optimal cost for periods } (r+1), \dots, n} \right\}$$

$x \geq 0$   
 $0 \leq i+x-d_r \leq H$

We reduce a period problem  $P$  to smaller (fewer periods) problem and use answers to solve  $P$ . We reduce stock problem to shortest path problem

