al

minimise $\underline{c}_1^T \underline{x}_1 + \underline{c}^T x + \underline{c}_3^T x_3$

s.t. $A_{1,1}\underline{x}_1 + A_{1,2}\underline{x}_2 + A_{1,3}\underline{x}_3 \geq \underline{b}_1$

$A_{2,1}\underline{x}_1 + A_{2,2}\underline{x}_2 + A_{2,3}\underline{x}_3 \leq \underline{b}_2$

$A_{3,1}\underline{x}_1 + A_{3,2}x_2 + A_{3,3}\underline{x}_3 = \underline{b}_3$

$\underline{x}_1 \geq 0, \ \underline{x}_2 \leq 0$

transpose $\Longrightarrow$

Dual

Maximise $\underline{b}_1^T \underline{y}_1 + \underline{b}_2^T \underline{y}_2 + \underline{b}_3^T \underline{y}_3$

s.t. $\underline{y}_1^T A_{1,1} + \underline{y}_2^T A_{2,1} + \underline{y}_3^T A_{3,1} \leq \underline{c}_1$

$\underline{y}_1^T A_{1,2} + \underline{y}_2^T A_{2,2} + \underline{y}_3^T A_{3,2} \geq \underline{c}_2$

$\underline{y}_1^T A_{1,3} + \underline{y}_2^T A_{2,3} + \underline{y}_3^T A_{3,3} = \underline{c}_3$

$\underline{y}_1 \geq 0, \ \underline{y}_2 \leq 0$

pes of constraint: Normal $\geq$ Anti-normal $\leq$ Tight $=$

ypes of variable: $\geq$ $\leq$ Unconstrained

Dual

$\underline{x}_1, \underline{x}_2, \underline{x}_3$

Primal Variables $\equiv$ Dual Constraints

Primal Constraints $\equiv$ Dual Variables

$\underline{y}_1, \underline{y}_2, \underline{y}_3$

$A_{i,j}$ are matrices

$\underline{x}_1, ---$ are vectors

D \ P	Solvable	Unbounded	Infeasible
Solvable	✓	✗	✗
unbounded	✗	✗	✓
infeasible	✗	✓	✓

Primal

$\rho_A = \max \ \xi$

Dual Variables

$q_1, q_2, \dots q_n$

$$\xi - \sum_{i=1}^{m} a_{ij} p_i \leq 0 \quad j = 1, 2, \dots, n$$

η

$$\sum_{i=1}^{m} p_i = 1$$

$$p_1, \dots, p_m \geq 0$$

$\Longrightarrow$ Dual

Dual

$\min . \ \eta$

$$q_1 + q_2 + \dots + q_n = 1 \quad \longleftarrow \text{constraint } \xi$$

$$\eta + \sum_{j=1}^{n} -a_{ij} q_j \geq 0 \quad \longleftarrow p_i$$

$$q_j \geq 0$$

ρ_B

$$\begin{array}{lcccc}
\max & 1 & 0 \ 0 \dots 0 & & \\
q_1 & 1 & \text{———} & \leq 0 \\
q_2 & 1 & \text{———} & \leq 0 \\
\vdots & 1 & \text{———} & \leq 0 \\
q_j & 1 & \text{—} a_{ij} \text{—} & \vdots \\
 & 1 & \text{———} & \\
 & 1 & \text{———} & \\
q_n & 1 & \text{———} & \leq 0 \\
\eta & & 1 \ 1 \ 1 \dots 1 & = 1 \\
 & \xi & p_1 \ p_2 \dots p_m &
\end{array}$$

Simple Aspects

Dominance

$$\begin{bmatrix} 3 & 5 & 7 & 9 & 4 & 6 \\ \cancel{2} & \cancel{4} & \cancel{6} & \cancel{9} & \cancel{3} & \cancel{5} \\ 4 & 6 & 2 & 5 & 3 & 6 \\ \cancel{2} & \cancel{7} & \cancel{5} & \cancel{4} & \cancel{3} & \cancel{3} \end{bmatrix}$$

Row $\underline{1}$ dominates Row 2

Col. 1 dominates col 6 & col 2 & col 4

Now

Row 1 dominates Row 4

$$\downarrow$$

$$\begin{bmatrix} 3 & 7 & 4 \\ 4 & 2 & 3 \end{bmatrix}$$

Non-Singular Game

Suppose A is non-singular and $\underline{1}^T A^{-1} \underline{1} \neq 0$

Value of game is $v = \dfrac{1}{\underline{1}^T A^{-1} \underline{1}}$ $\qquad \underline{1} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

provided

$$\underline{p} = v \, \underline{1}^T A^{-1}, \quad \underline{q} = v A^{-1} \underline{1} \geq 0$$

$\underline{p}$ is feasible for primal

$\underline{q}$ is " " dual

objective values are equal to v.

Symmetric Games

$$A^T = -A$$

Value of game $= 0$

$$\underline{p}^T A \underline{p} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} p_i p_j = \sum_i a_{ii} p_i + \sum_{i \neq j} (a_{ij} + a_{ji}) p_i p_j$$

$$a_{ii} = -a_{ii} = 0$$

$$a_{ij} + a_{ji} = 0, \; i \neq j$$

$$P_A(\underline{p}, \underline{q}) \quad \begin{bmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{bmatrix} \longleftarrow$$

$$P_A \leq 0 \quad \& \quad P_A = P_B \Longrightarrow P_A = P_B = 0$$

$$P_B \geq 0$$