

minimize $c_1^T x_1 + c_2^T x_2 + c_3^T x_3$
 s.t. $A_{11}x_1 + A_{12}x_2 + A_{13}x_3 \geq b_1$
 $A_{21}x_1 + A_{22}x_2 + A_{23}x_3 \leq b_2$
 $A_{31}x_1 + A_{32}x_2 + A_{33}x_3 = b_3$
 $x_1 \geq 0, x_2 \leq 0$

Dual

Maximize $b_1^T y_1 + b_2^T y_2 + b_3^T y_3$
 s.t. $y_1^T A_{11} + y_2^T A_{21} + y_3^T A_{31} \leq c_1$
 $y_1^T A_{12} + y_2^T A_{22} + y_3^T A_{32} \geq c_2$
 $y_1^T A_{13} + y_2^T A_{23} + y_3^T A_{33} = c_3$
 $y_1 \geq 0, y_2 \leq 0$

transpose \Rightarrow

type of constraint: Normal \geq , Anti-normal \leq , Tight $=$
 type of variables: \geq , \leq , Unconstrained

Dual \updownarrow
 x_1, x_2, x_3
 Primal Variables \equiv Dual Constraints
 Primal Constraints \equiv Dual Variables
 y_1, y_2, y_3

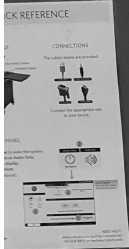
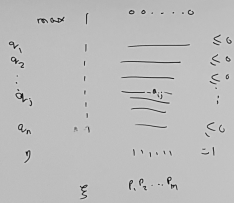
$A_{i,j}$ are matrices

x_1, \dots are vectors

| P | Solvable | Unbounded | Infeasible |
|------------|----------|-----------|------------|
| D Solvable | ✓ | ✗ | ✗ |
| unbounded | ✗ | ✗ | ✓ |
| Infeasible | ✗ | ✓ | ✓ |

$P_A = \max \sum_{i=1}^m c_i P_i$
 Dual Variables q_1, q_2, \dots, q_n
 $\sum_{i=1}^m a_{ij} P_i \leq 0 \quad j=1, 2, \dots, n$
 $\sum_{i=1}^m P_i = 1$
 $P_1, \dots, P_m \geq 0$

Dual $M \text{ min } \gamma$
 $q_1 + q_2 + \dots + q_n = 1$ ← constraint S
 $\gamma + \sum_{j=1}^n -a_{ij} q_j \geq 0$ ← P_i
 $q_j \geq 0$ ← P_B



Simple Aspects

Dominance

$$\begin{bmatrix} 3 & 5 & 7 & 9 & 4 & 6 \\ 2 & 4 & 6 & 9 & 3 & 5 \\ 4 & 6 & 2 & 5 & 3 & 6 \\ 2 & 7 & 5 & 4 & 3 & 3 \end{bmatrix}$$

Row 1 dominates Row 2

Col 1 dominates Col 6 & col 2 & col 4

Now

Row 1 dominates Row 4

↓

$$\begin{bmatrix} 3 & 7 & 4 \\ 4 & 2 & 3 \end{bmatrix}$$

Non-Singular Game

Suppose A is non-singular and $\underline{1}^T A^{-1} \underline{1} \neq 0$

Value of game is $v = \frac{1}{\underline{1}^T A^{-1} \underline{1}} \underline{1} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$

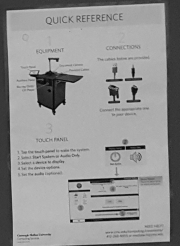
provided

$$\underline{p} = v \underline{1}^T A^{-1}, \underline{q} = v A^{-1} \underline{1} \geq 0$$

\underline{p} is feasible for primal

\underline{q} is " " " dual

Objective values are equal to v .



Symmetric Games

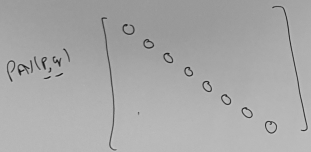
$$A^T = -A$$

Value of game = 0

$$\underline{P}^T A \underline{P} = \sum_{i=1}^n \sum_{j=1}^n a_{ij} p_i p_j = \sum_i a_{ii} p_i + \sum_{i \neq j} (a_{ij} + a_{ji}) p_i p_j$$

$$a_{ii} = -a_{ii} = 0$$

$$a_{ij} + a_{ji} = 0, i \neq j$$



$$P_A \leq 0 \quad \& \quad P_A = P_B \Rightarrow P_A = P_B = 0$$
$$P_B \geq 0$$