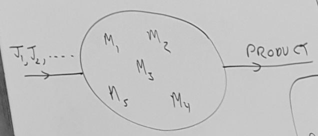
Machine Scheduling

Machine Shop



Example 1

$$1 \mid \dots \mid \sum_j \omega_j C_j$$

# of machines

restrictions, comments

Objective

Weighted Completion time

Problem: which order of jobs minimises the objective function?

C_j : completion time of job j.

P_j = processing time.

ω_j = weight.

P_1	P_2
2	3

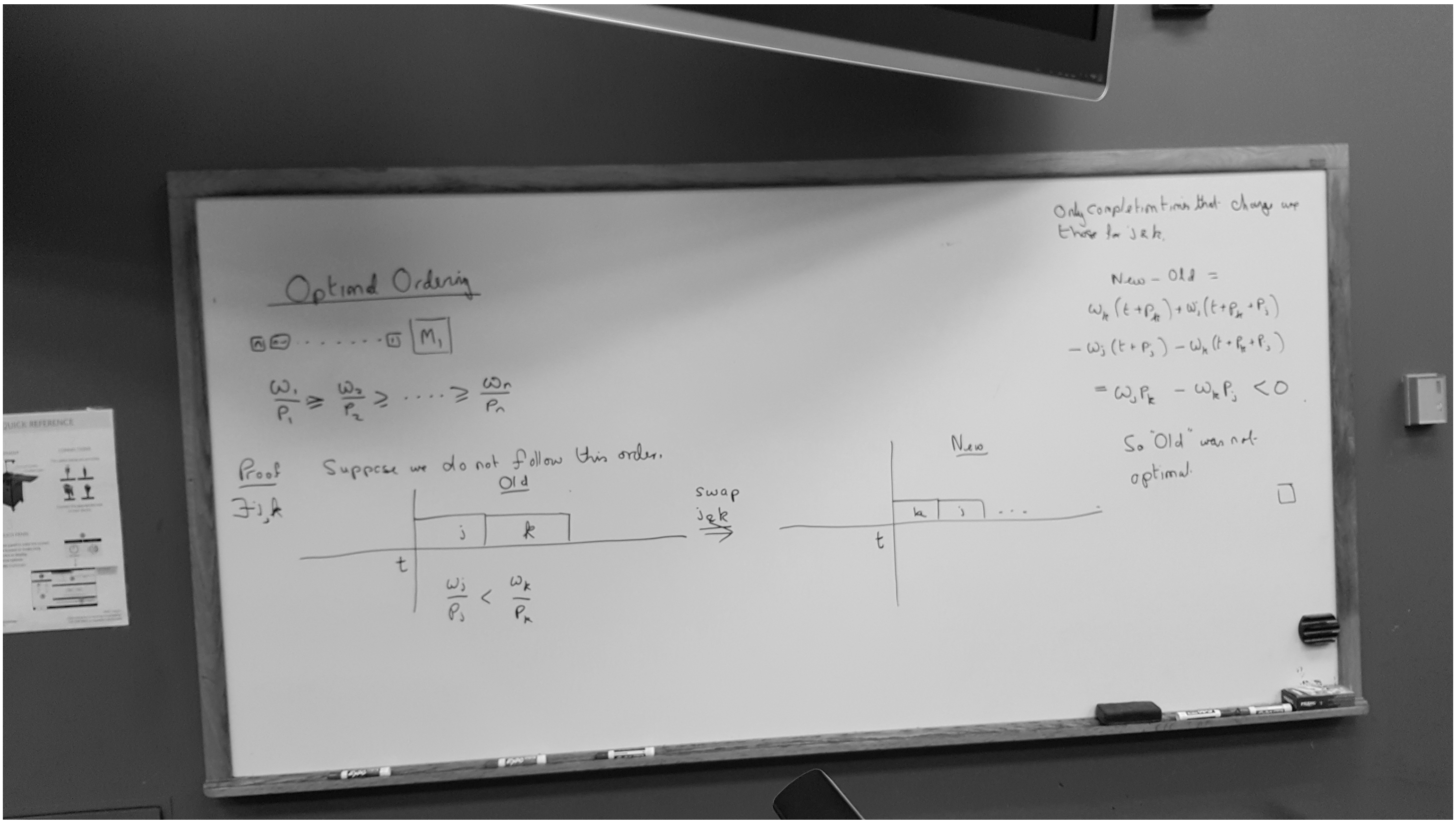
$\underbrace{2}+\underbrace{2+3}$

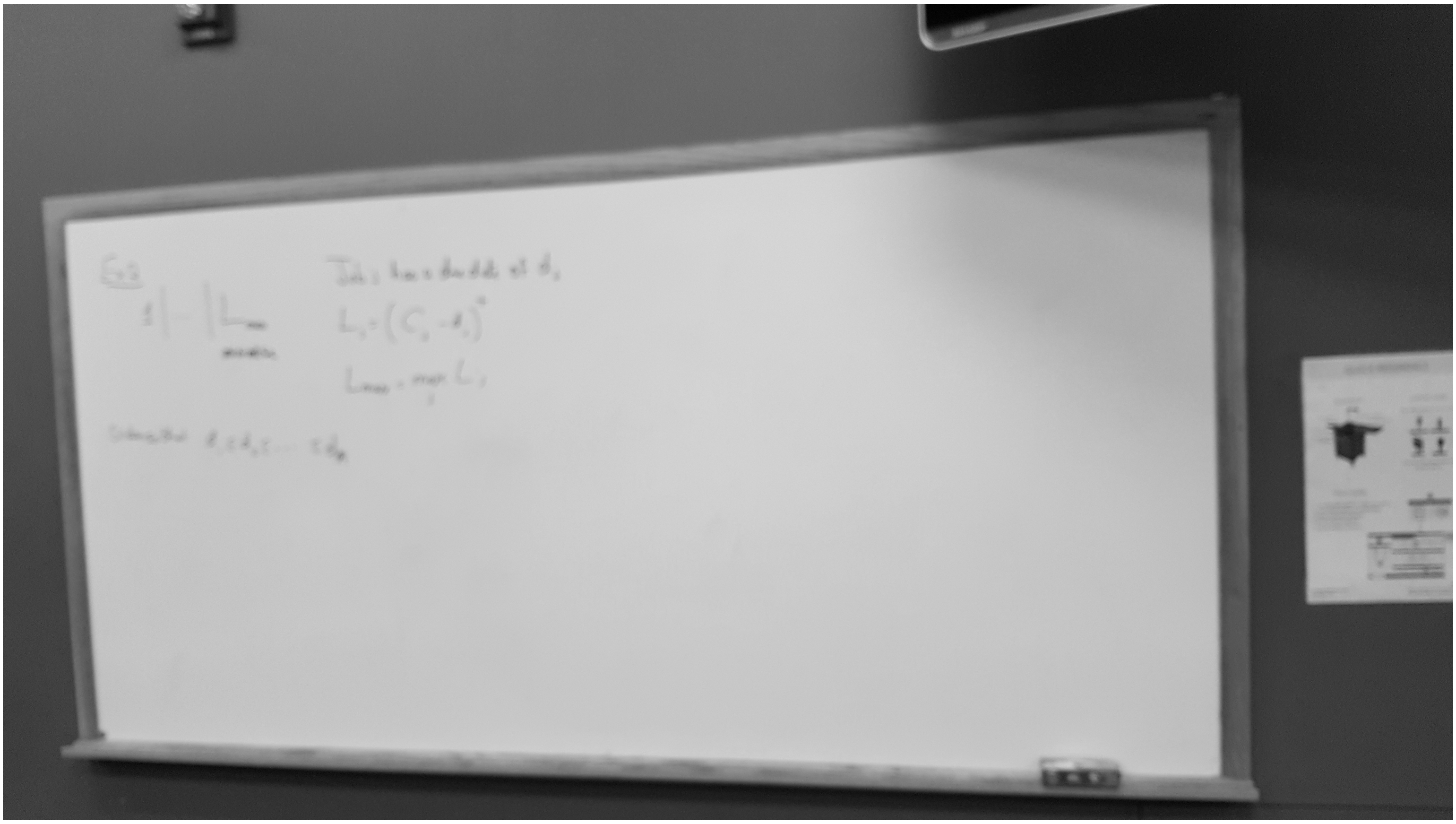
$3+3+2$

$\frac{\omega_1}{P_1}$

Proof

$\exists j, k$


Optimal Ordering
n n-1 . . . . . . 1 M1
ω1/P1 ≥ ω2/P2 ≥ . . . . ≥ ωn/Pn
Proof
∃ j, k
Suppose we do not follow this order.
Old
j
k
t
ωj/Pj < ωk/Pk
swap
j&k
New
k
j
t
Only completion times that change are those for j&k.
New − Old =
ωk(t + Pk) + ωj(t + Pk + Pj)
− ωj(t + Pj) − ωk(t + Pk + Pj)
= ωjPk − ωkPj < 0
So "Old" was not optimal.



Optimal Ordering

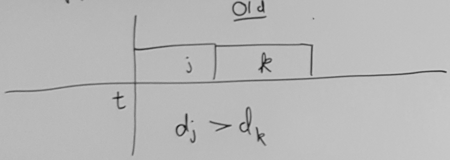
$d_1 \le d_2 \le \cdots \le d_n$

Proof

$\exists j, k$

Suppose we do not follow this order.

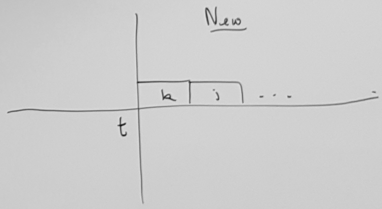
Old



$d_j > d_k$

swap $j \& k$ $\Rightarrow$

New



Only completion times that change are those for $j \& k$.

Contribution to objective of $j \& k$.

Old: $\max\left\{ \left(C_j^{old} - d_j\right)^+, \left(C_k^{old} - d_k\right)^+ \right\}$

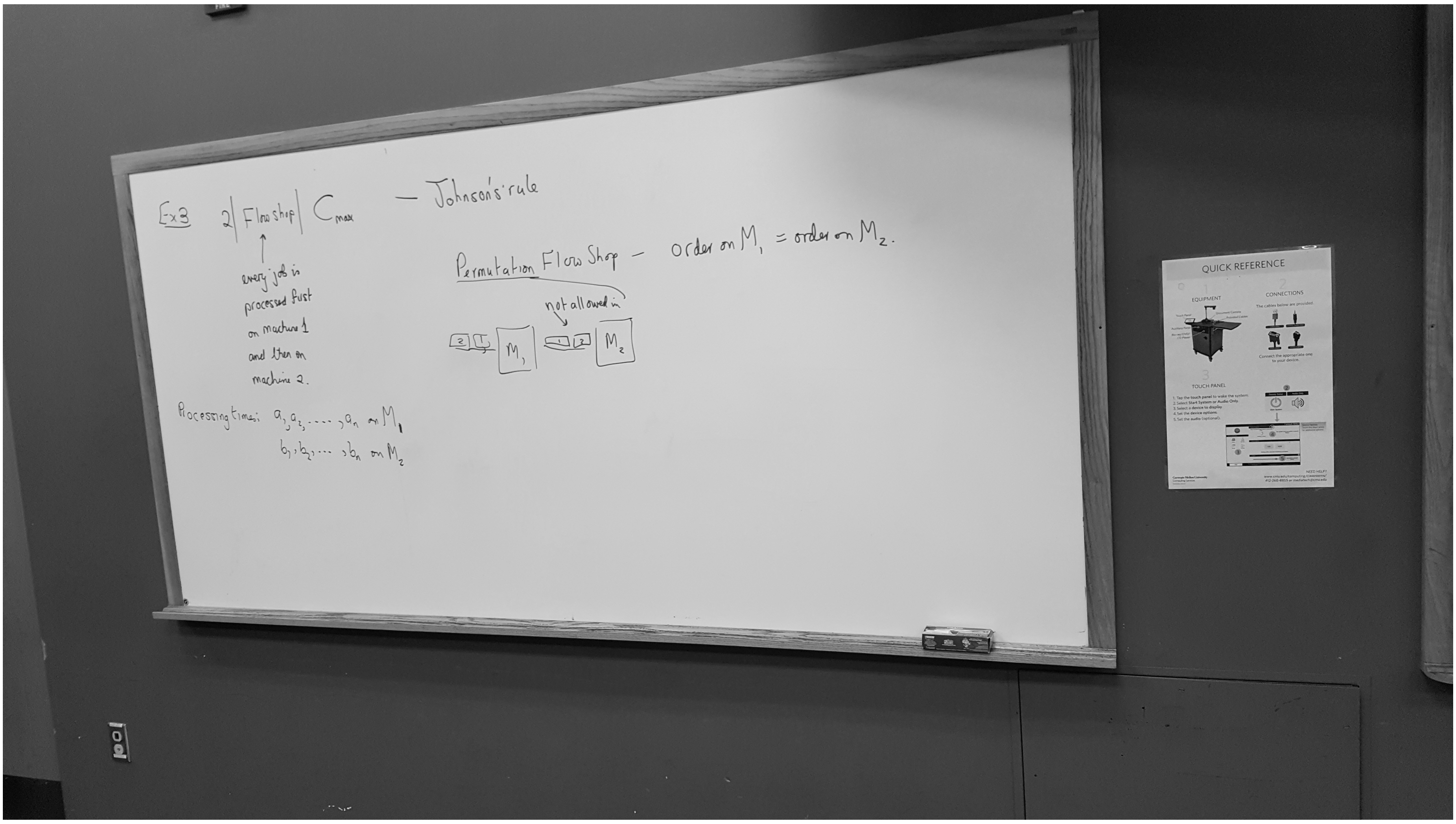
$= \left(C_k^{old} - d_k\right)^+$

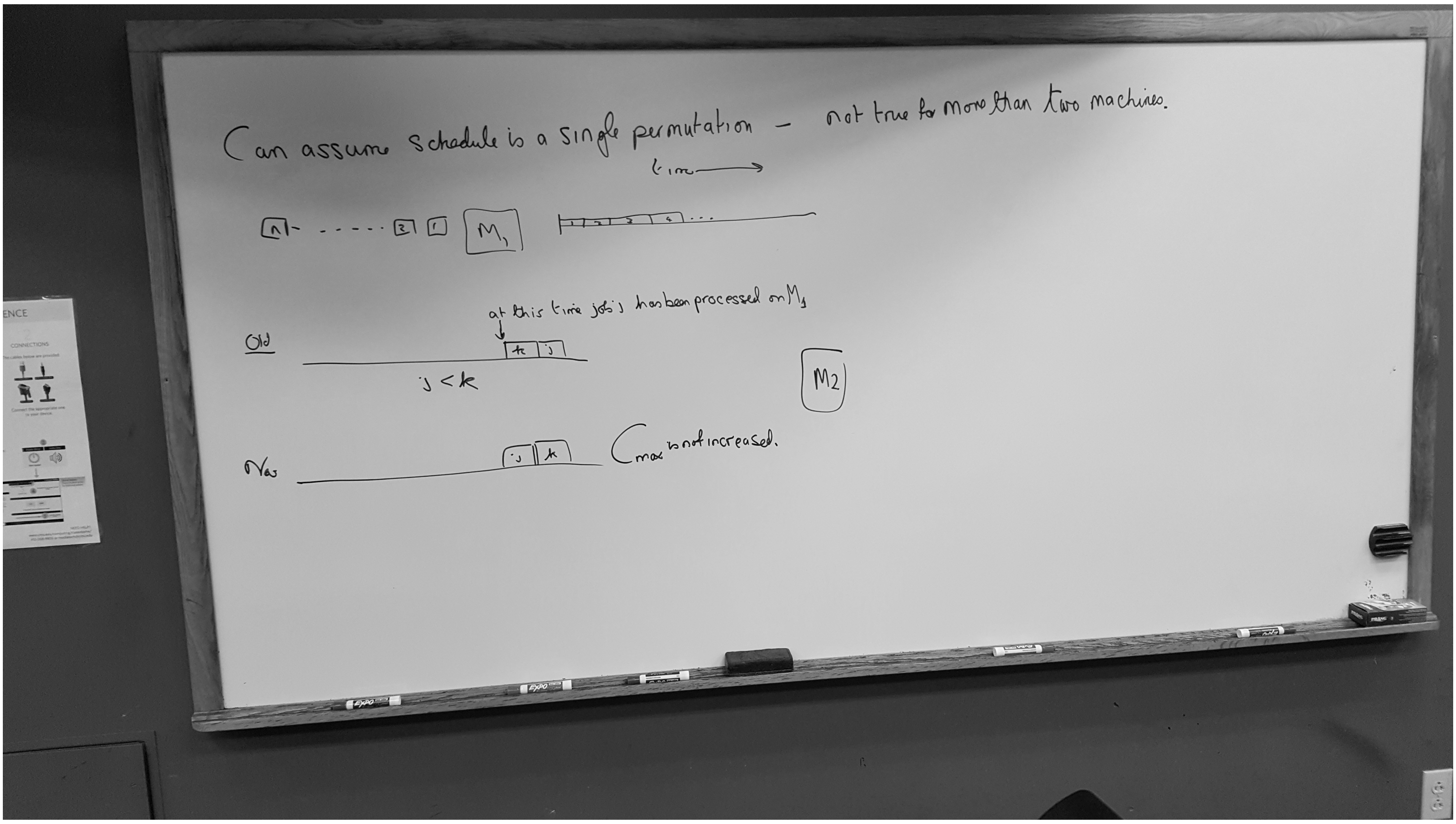
New: $\max\left\{ \left(C_j^{new} - d_j\right)^+, \left(C_k^{new} - d_k\right)^+ \right\}$

$\le \left(C_k^{old} - d_k\right)^+$ $\quad C_k^{new} \le C_k^{old}$

So new $\le$ old

$\square$


Ex3 2 | Flow shop | C_{max} — Johnson's rule
every job is processed first on machine 1 and then on machine 2.
Permutation Flow Shop — order on M_1 = order on M_2.
not allowed in
2 1 M_1 1 2 M_2
Processing times: $a_1, a_2, \ldots, a_n$ on M_1
$b_1, b_2, \ldots, b_n$ on M_2
QUICK REFERENCE
EQUIPMENT
CONNECTIONS
TOUCH PANEL


Can assume schedule is a single permutation – not true for more than two machines.
time
n
2
1
M1
1
2
3
4
...
at this time job j has been processed on M1
Old
k
j
j < k
M2
New
j
k
Cmax is not increased.