Multi-Item Models

Model 4

n distinct types of item.

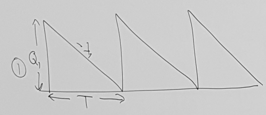
Demand for item j is λ_j

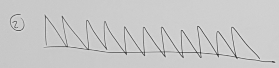
Inventory cost is I_j

Cost of making an order A is independent of the set and number of items ordered.

Observation

Best to order all things at same times





Ordering ① at same time as ②, reduces order cost and inventory cost.

We just have to determine T (time between orders)

Order Q_j of item j where $Q_j = \lambda_j T$

Total cost per period

$$K = \underbrace{\frac{A}{T}}_{\text{order cost}} + \underbrace{\left(\frac{1}{2}\sum_{j=1}^{n} I_j \lambda_j\right) T}_{\text{Inventory cost}}$$

Solve for T: $\frac{dK}{dT} = 0$.

"W = Wilson"

Model 5

Suppose we impose the constraint

$$\sum_{j=1}^{n} f_j Q_j \le f$$

f_j could be warehouse space of item j.

cost of ordering item j

$$K = \sum_{j=1}^{n} \left[\lambda_j A_j + \tfrac{1}{2} I_j Q_j \right]$$

Solve minimise $K(Q_1, Q_2, \dots Q_n)$

subject to

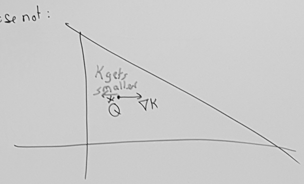
(*) $f_1 Q_1 + f_2 Q_2 + \dots + f_n Q_n \le f$

$Q_1, Q_2, \dots, Q_n \geq 0$

Claim: in Case (ii) optimum Q^* satisfies

$f_1 Q_1^* + f_2 Q_2^* + \dots + f_n Q_n^* = f$

Suppose not:



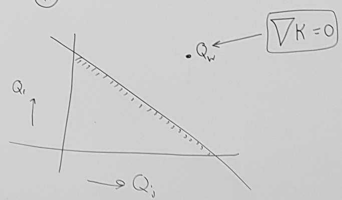
(i) Compute optimum, ignoring (*)

i.e. $\frac{\partial K}{\partial Q_j} = 0 \Rightarrow Q_{j,W} = \sqrt{\frac{2\lambda_j A_j}{I_j}}$

(i) $\underline{Q}_W = (Q_{1,W}, Q_{2,W}, \dots, Q_{n,W})$ satisfies (*)

Done

(ii) (*) is not satisfied.

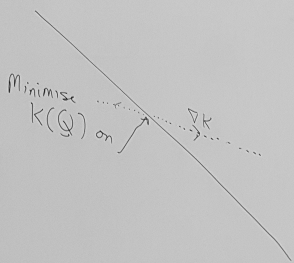


We have to solve:

minimise $K(\underline{Q})$

s.t. $\sum_{j=1}^{n} f_j Q_j = f$

$\boxed{\text{Ignore } Q_j \geq 0 \text{ for now.}}$



<u>Claim</u>: $\exists \theta : \nabla K = \theta (f_1, f_2, \ldots, f_n)$

θ is the <u>Lagrange multiplier</u>

Find θ.

$$\frac{\partial K}{\partial Q_j} = \theta f_j \quad \text{or} \quad -\frac{\lambda_j A_j}{Q_j^2} + \frac{1}{2} I_j = \theta f_j$$

(a) Solve for $Q_j = Q_j(\theta) = \sqrt{\dfrac{2\lambda_j A_j}{I_j - 2\theta f_j}}$

(b) Solve for θ: $\sum_{j=1}^{n} f_j Q_j(\theta) = f$

numerically

Assume that $I_j - 2\theta f_j > 0, \ \forall j$