Inventory Control

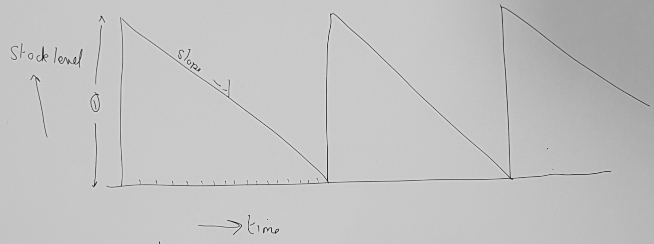
Model 1

Demand for product is λ per period.

Cost of making an order is A [independent of quantity ordered]

Cost I per period of keeping one unit in stock.

Parameters: T = time between orders = T periods
Q = quantity ordered



K

$$\text{Cost per period period} = \underbrace{\frac{A}{T}}_{\text{Order cost}} + \underbrace{\frac{IQ}{2}}_{\text{Inventory Cost}}$$

$$Q = \lambda T$$

$$K = \frac{A\lambda}{Q} + \frac{IQ}{2}$$

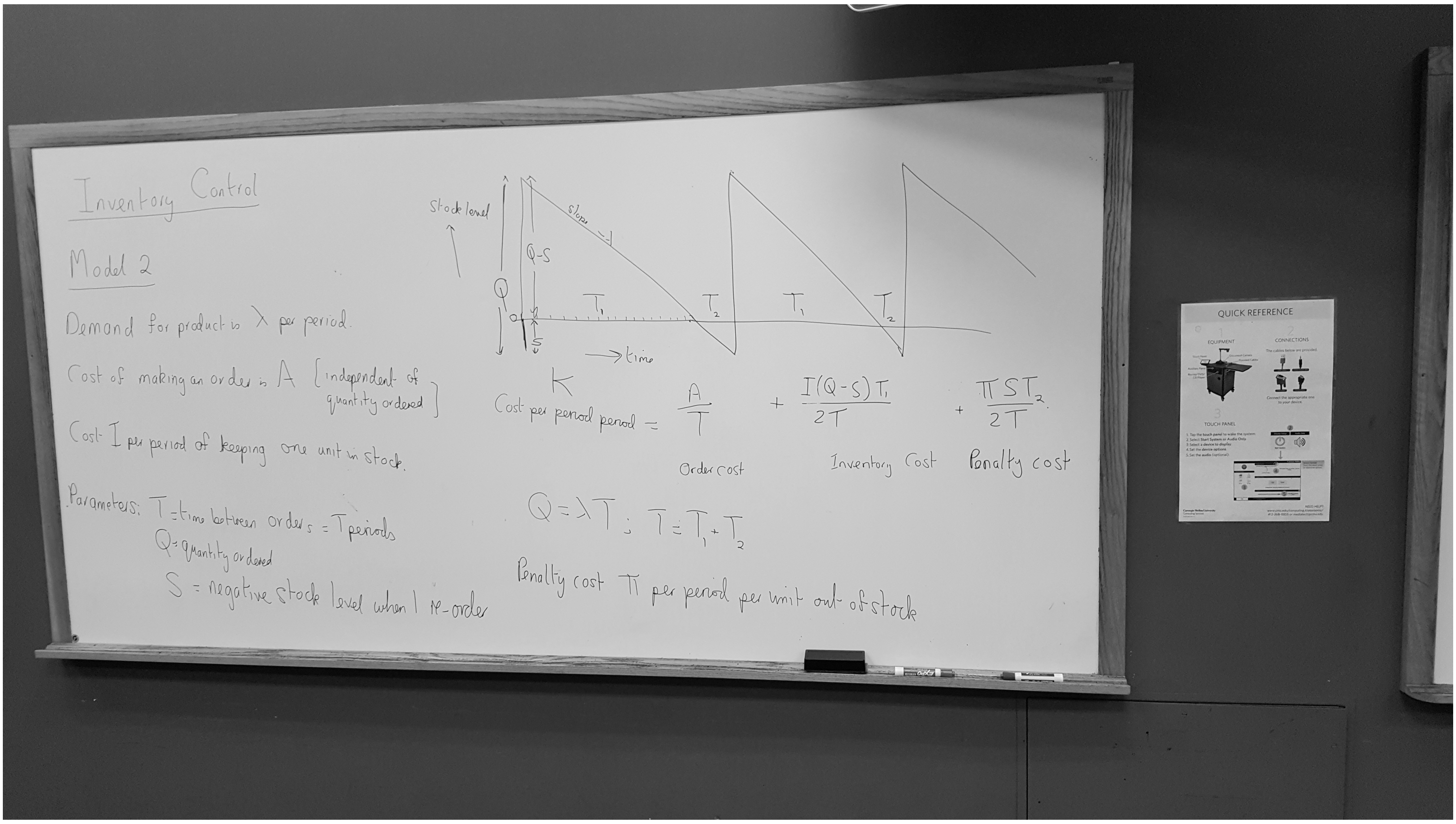
$$\frac{dK}{dQ} = -\frac{A\lambda}{Q^2} + \frac{I}{2}$$

$= 0$ when

$$Q = Q_W = \sqrt{2\lambda A / I}$$

$$T_W = \sqrt{2A / \lambda I}$$

$$K_W = \sqrt{2\lambda A I}$$


Inventory Control
Model 2
Demand for product is λ per period.
Cost of making an order is A [independent of quantity ordered]
Cost I per period of keeping one unit in stock.
Parameters: T = time between orders = T periods
Q = quantity ordered
S = negative stock level when I re-order
Stock level
slope
Q
Q-S
0
S
T1
T2
T1
T2
time
K
Cost per period period = A/T + I(Q-S)T1/2T + πST2/2T
Order cost
Inventory Cost
Penalty cost
Q = λT ; T = T1 + T2
Penalty cost π per period per unit out of stock
QUICK REFERENCE

$$\left.\begin{aligned} Q &= \lambda T \\ Q - S &= \lambda T_1 \\ S &= \lambda T_2 \end{aligned}\right\} \text{ Can write } K = K(Q, S)$$

$$= \frac{\lambda A}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

Put $\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$ and solve.

Solution: $S = \sqrt{\frac{2\lambda A I}{\pi(\pi + I)}}$; $Q = Q_W \sqrt{\frac{\pi + I}{\pi}}$; $K = K_W \sqrt{\frac{\pi}{\pi + I}}$.

Inventory Control

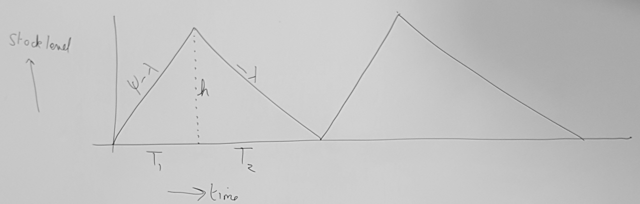
Model 3

Demand for product is λ per period.

Cost of making an order is A [independent of quantity ordered]

Cost I per period of keeping one unit in stock.

Parameters: T = time between orders = T periods
Q = quantity ordered



K
Cost per period period $= \dfrac{A}{T} + \dfrac{Ih}{2}$

Order cost — Inventory Cost

$Q = \lambda T$; $T = T_1 + T_2$

When you make an order it arrives at rate $\psi \geq \lambda$.

$$\left.\begin{aligned} Q &= \lambda T \\ T &= T_1 + T_2 \\ (\psi - \lambda) T_1 &= \lambda T_2 = h \end{aligned}\right\} \text{eliminate } T, T_1, T_2, h$$

$$K = \frac{A\lambda}{Q} + \frac{1}{2} I Q \left[1 - \frac{\lambda}{\psi} \right]$$

$$\frac{dK}{dQ} = 0 : \qquad Q = Q_w \sqrt{\frac{\psi}{\psi - \lambda}} \; ; \qquad K = K_w \sqrt{\frac{\psi - \lambda}{\psi}}$$