9/9/15

Knapsack Problem

$$f_r(w) = \max_{0 \le x \le \left\lfloor \frac{w}{w_r} \right\rfloor} \left[c_r x + f_{r-1}(w - w_r x_r) \right]$$

maximum value from items 1, 2, .. r into a knapsack of size w

Using this recurrence takes $O(nW^2)$ "time".

We can reduce this to $O(nW)$.

Another recurrence:

$$f_r(w) = \max \begin{cases} f_{r-1}(w) : & \text{no type } r \\ c_r + f_r(w - w_r) : & \text{at least one type } r. \end{cases}$$

Requires $O(nW)$ "time".

1,3,6,8 & 2,2,3,3 ;

Maximise $x_1 + 3x_2 + 6x_3 + 8x_4$ $0 \le x_1, \ldots$ integer

s.t. $2x_1 + 2x_2 + 3x_3 + 3x_4 \le 10$

w	f_1	b_1	f_2	b_2		f_3	b_3	f_4	b_4	
0	0	0	0	0		0	0	0	0	
1	0	0	0	0		0	0	0	0	
2	[illegible]	1	3	1		3	0	3	0	
3	1	1	3	1		6	1	8	1	
4	2	1	6	1		6	0/1	8	1	
5	2	1	6	1		9	1	11	1	
6	3	1	9	1		12	1	16	1	
7	3	1	9	1		12	1	16	1	
8	4	1	12	1		15	1	19	1	
9	4	1	12	1		18	1	24	1	
10	5	1	15	1		18	1	24	1	

$x_4 = 3$

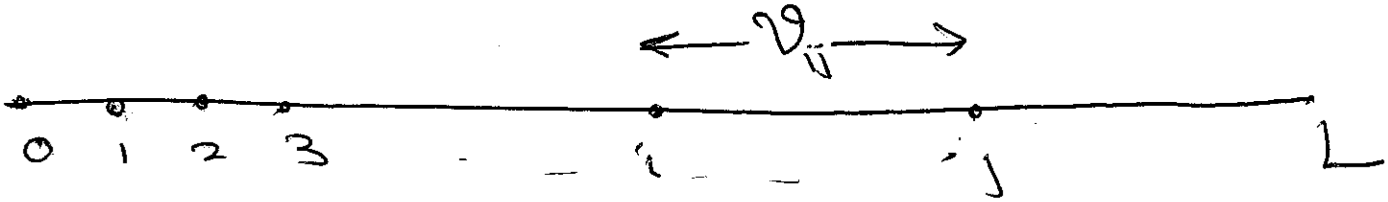
$x_1 = x_2 = x_3 = 0$

Minimise $2x_1 + 2x_2 + 3x_3 + 3x_4$ ← weight.

s.t. profit $= x_1 + 3x_2 + 6x_3 + 8x_4 \geq 18$?

Answer 8

Breaking up a stick



Value of stick [i, i+1, ... j] is v_{ij}

Problem: break up the stick to maximize the value.

$f(l)$ = maximum obtainable from [0, l]

$$= \max_{0 \leq x < L} \left[v_{x,L} + f(x) \right]$$

$f(0) = 0$

Algorithm is $O(L^2)$