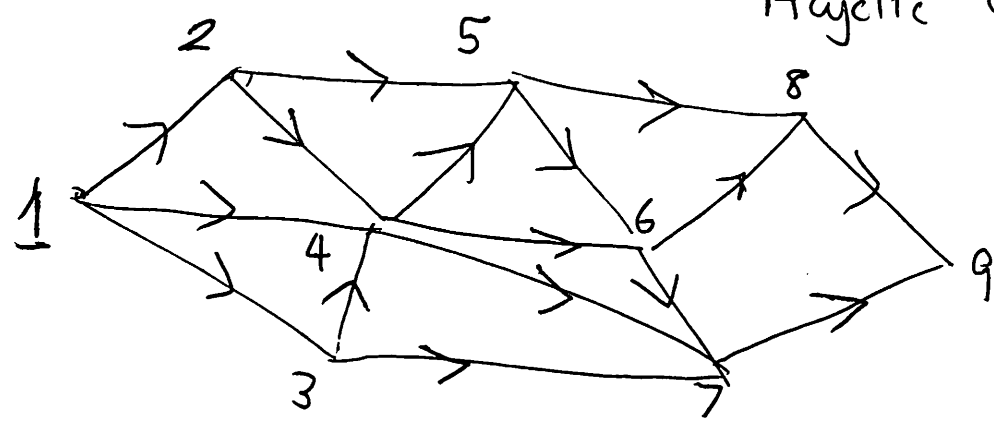
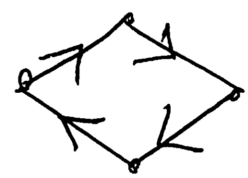
9/30/15

Longest path in a DAG [Directed Acyclic Graph]



No directed cycles



Topological Ordering/Numbering

DAG $G = (V, A)$: label V from 1 to n such that

$|V| = n$

$f: V \to [n]$

$$i \longrightarrow j$$

$$i < j$$

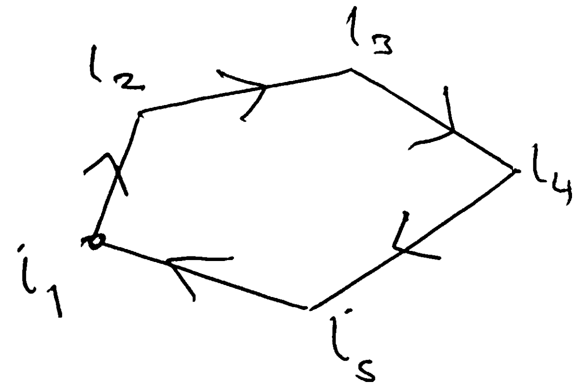
Theorem

A Digraph G is a DAG $\Leftrightarrow$ if it has a topological ordering

Proof

Suppose G has a topological ordering.

No directed cycles:



$i_1 < i_2 < i_3 < i_4 < i_5 < i_1$

Suppose G is a DAG.

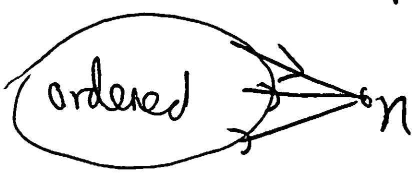
G has at least one sink - vertex with no outgoing arcs

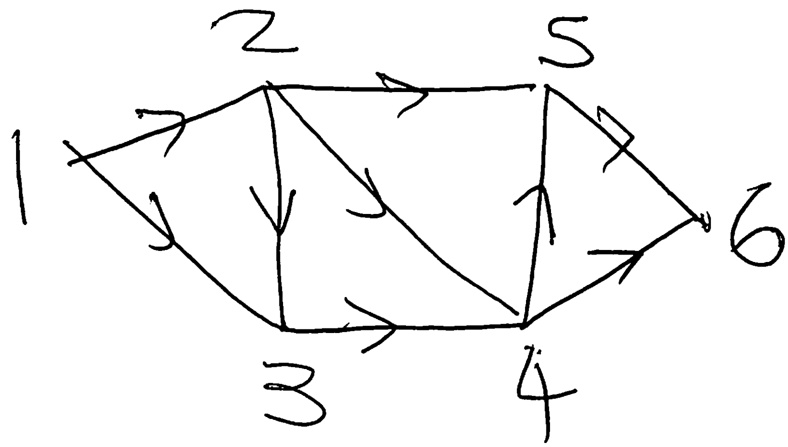
Let $P = (x_1, x_2, \ldots, x_m)$ be a longest directed path.

x_m is a sink

Algorithm: (i) Find a sink w - label it n

(ii) Topologically order $G \setminus \{n\}$ - induction says we can do.





Suppose we want to find a longest path from 1 to every other vertex.

Optimality Condition: $d(j)$ = length of path to j.

$$d(j) = \max_{i<j} d(i) + \ell(i,j)$$

$d(1) = 0$

$d(2) = \max\{ d(1) + \ell(1,2) \}$

linear time algorithm

$d(3) = \max\{ d(1) + \ell(1,3),\ d(2) + \ell(2,3) \}$

$\vdots$

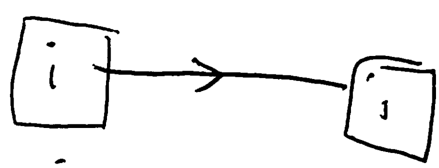
Project Scheduling – Critical Path Method

A project consists of n jobs – activities

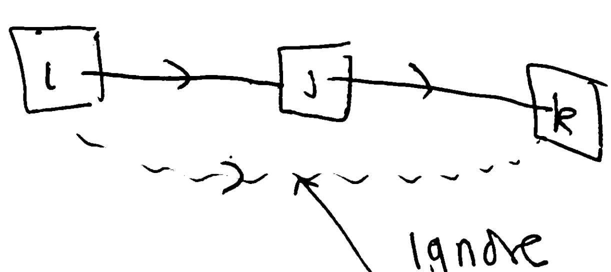
Making a cup of tea:

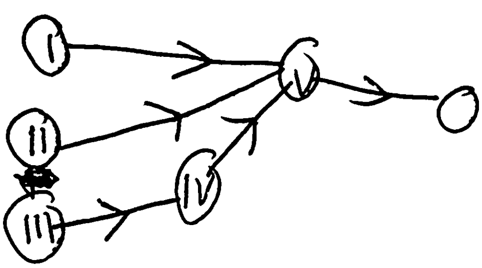
(i) Get a cup from cupboard
(ii) Get a tea bag
(iii) Fill the kettle with water
(iv) Boil the water
(v) Pour water into cup
(vi) Allow to brew.

Define a digraph:

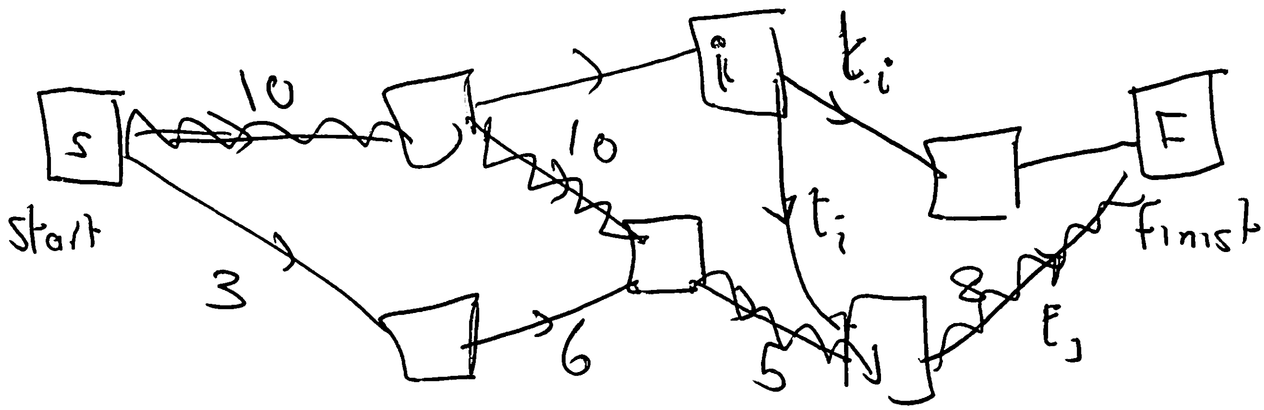


i must finish before j





Associate a time t_i to complete activity i.



Critical Path = longest $\boxed{s} \rightarrow \boxed{F}$ path