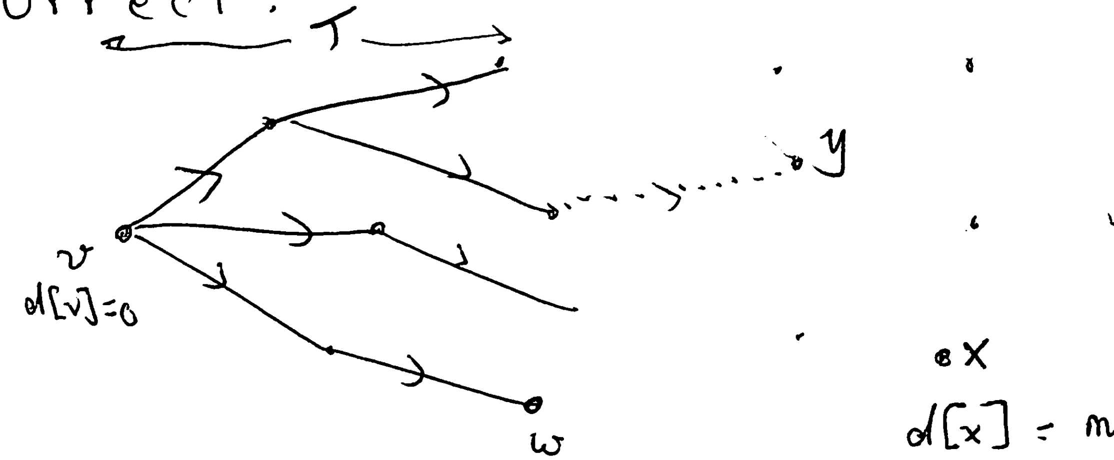
9\25\15

Shortest path $\ell(e) \geq 0$.

Proof that Dijkstra algorithm is correct:



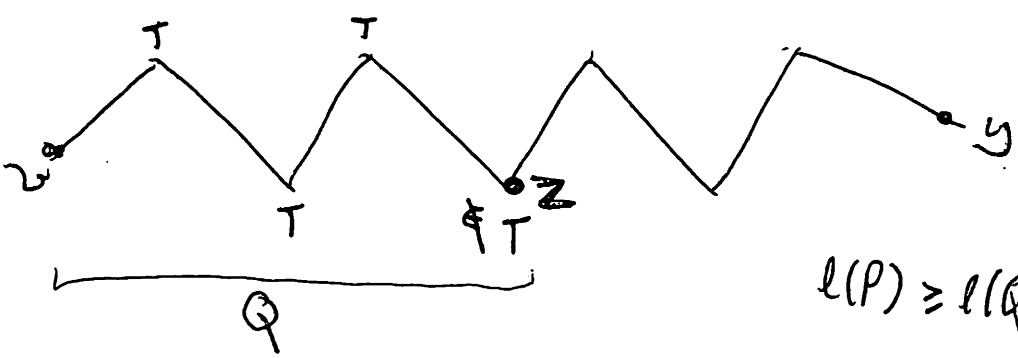
$d[w] =$ min. length path $v \longrightarrow w$.

$d[x] =$ min. length path $v \longrightarrow x$ in which ~~only~~ final edge is not in T.

This set up holds initially with $T = \{v\}$.

Now add y to T where $d[y]$ minimises $d[x]$ for $x \notin T$.

Need only show that $d[y]$ is correct.



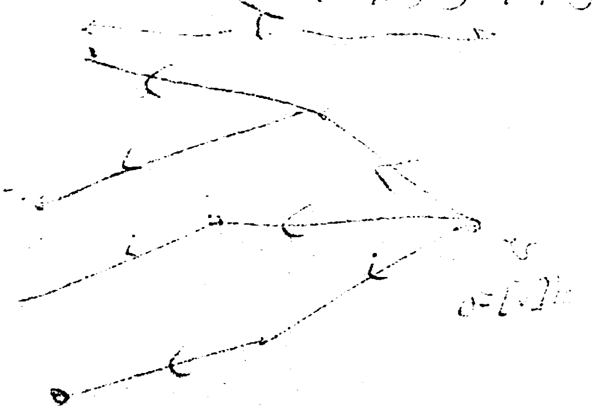
$\ell(P) \geq d[y]$

$\ell(P) \geq \ell(Q) \geq d(z) \geq d[y]$

9/25/15

Shortest path $l(e) \geq 0$.

Proof that Dijkstra algorithm is correct:



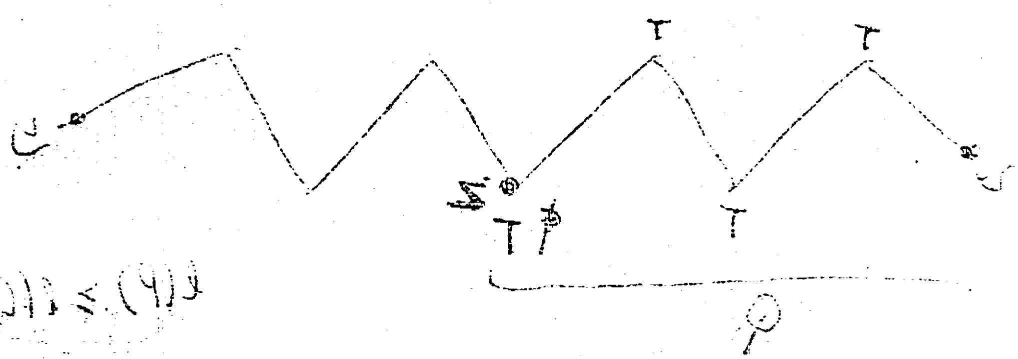
x — $d[x]$ = min length path $v \to x$ within edges of T

w — $d[w]$ = min length path $s \to w$

This set up holds initially with $T = \{s\}$

we add y to T when $d[y]$ minimizes $d[x]$ for $x \notin T$

Need only show that $d[y]$ is correct.

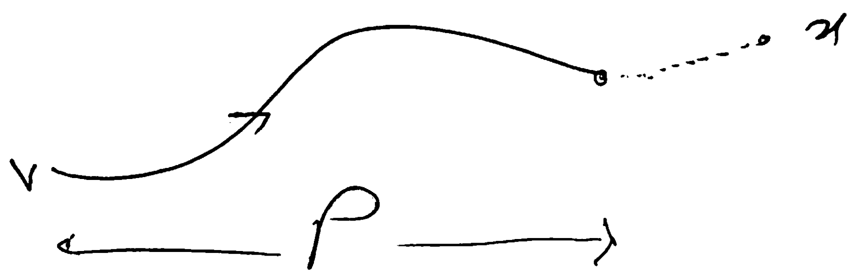


$l(p) \geq d[y]$

$l(p) \geq l(P) \geq d(z) = d[y]$

Suppose I change the definition of l and I ensure that

$$l(P; x) \geq l(P)$$

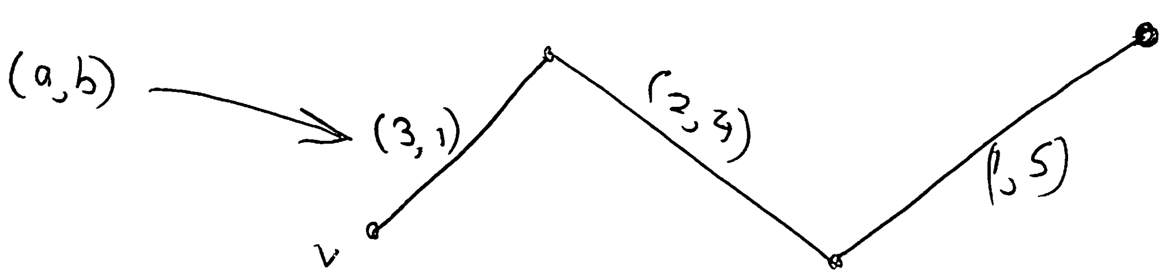


<u>Possible example</u>

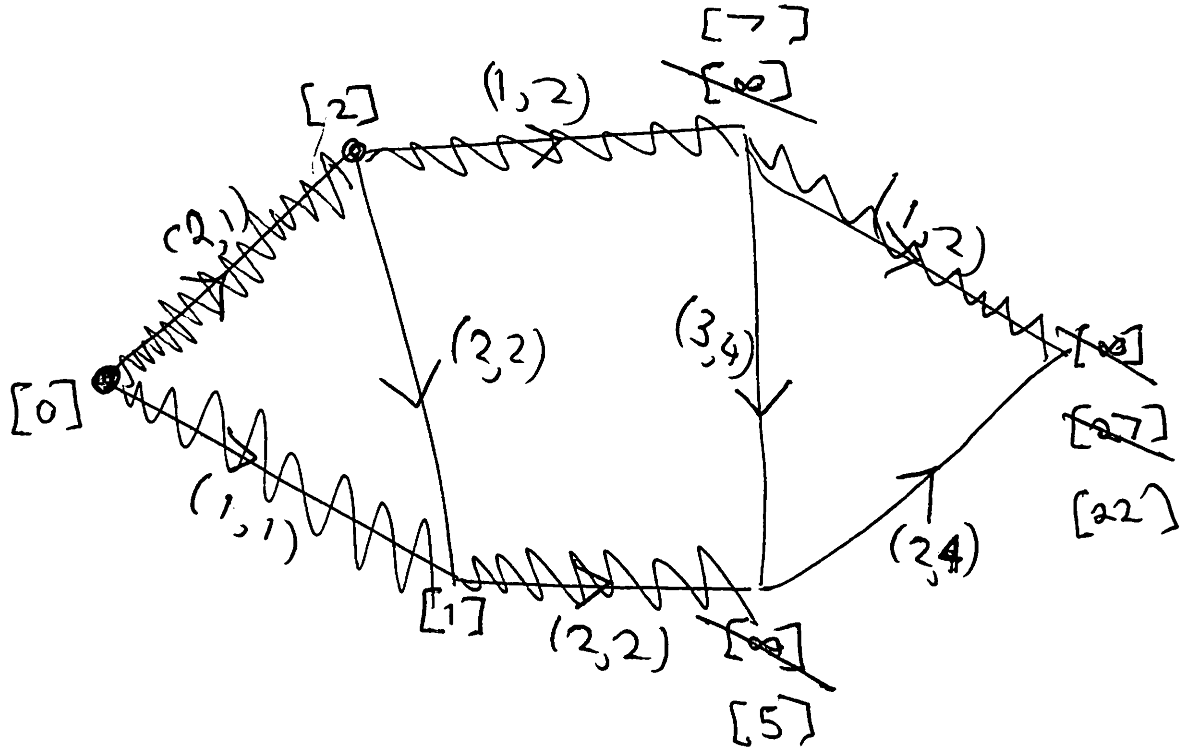
Length of an edge could depend on the time of day.

$$l(e = (x, y)) = \underset{\geq 0}{a_e} + \underset{\geq 0}{b_e}\, t$$

where t = time to get from v to x.



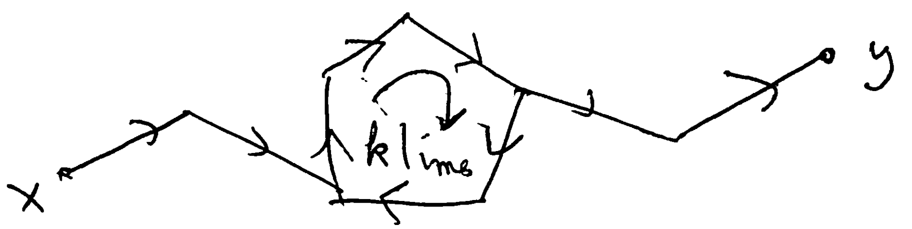
$$l(P) = 3 + 14 + 86 = 103$$



[7]
[∞]
(1,2)
[2]
(2,1)
(1,2)
(2,2)
(3,4)
[0]
[∞]
[27]
(1,1)
(2,4)
[22]
[1]
(2,2)
[∞]
[5]

Next we allow negative arc lengths.

Suppose $\exists$ a cycle C such that $l(C) < 0$.

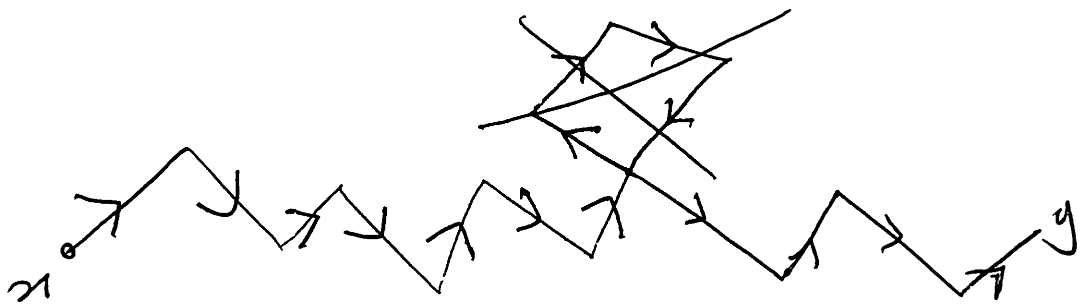


$$l(\text{Walk}) = c_1 + k\,l(C) \longrightarrow -\infty \text{ with } k.$$

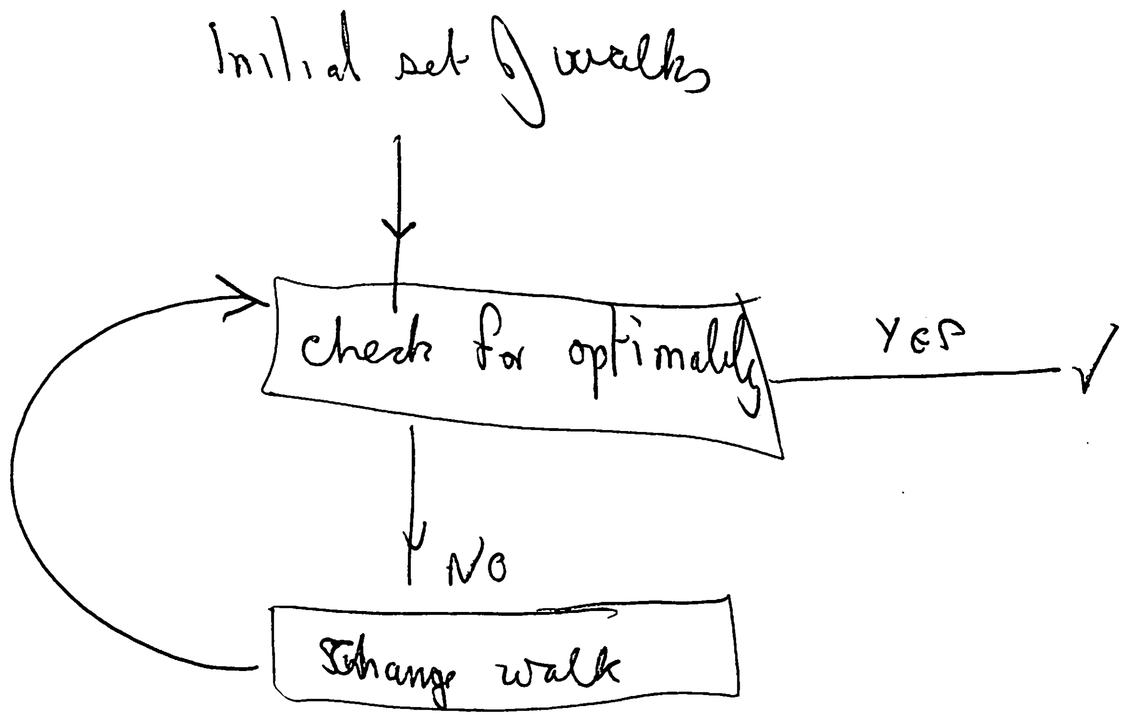
I.e. there is no shortest walk from x to y.

We do not consider cases where $l(C) < 0$ for some cycle.

If $l(C) \geq 0$ for all C then there is a shortest path that is also a shortest walk.

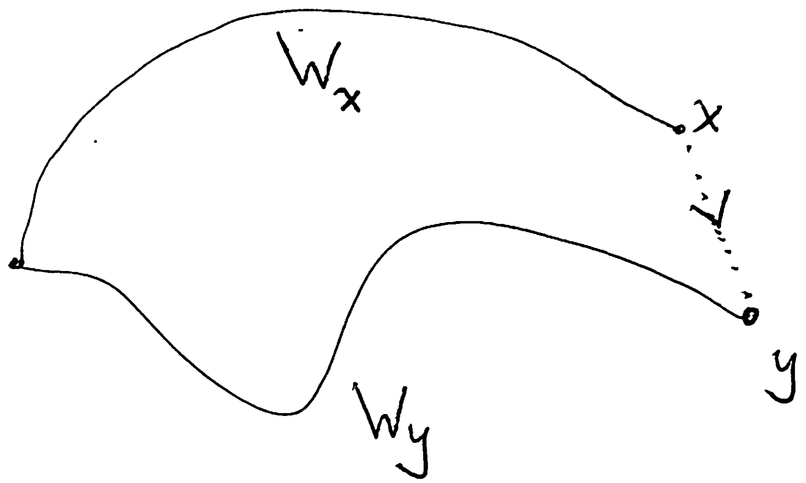


A shortest walk with fewest edges is a path.



Suppose we have a set of walks W_x from s to every other vertex x.

$$d[x] = \ell(W_x).$$



If $d[x] + \ell(x,y) < d(y)$ then W_y is not minimal.

Optimality Condition

$$d[y] \leq d[x] + \ell(x,y), \quad \forall \text{ edges } (x,y).$$