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Production model with random demand: $P[d_r = d] = p_d$

(I) Demand known before choose production level

min expected cost

$$f_r(i) = \sum_{d=0}^{\infty} p_d \min_x \left[c(x) + f_r(i + x - d) \right]$$

(II) Demand known after production level is set. Penalty π per unit of unmet demand

$$f_r(i) = \min_x \left[c(x) + \sum_d p_d \left([d - x - i]^+ \pi + f_{r+1}([i + x - d]^+) \right) \right]$$

Infinite Planning Horizon

Suppose demand = d, n = 1, 2, ...

Suppose a policy gives costs

Policy 1   4, 4, 4, 4, 4, 4, 4, ...

Policy 2   3, 3, 3, 3, 3, 3, 3, ...

Policy 3   2, 4, 2, 4, 2, 4, 2, ...

We can't try to minimize total cost!

Policy 2 & 3 have better average cost than policy 1.

We will have a discount factor $\alpha < 1$ and \$1 in period $i+1$ is the "same" as $\$\alpha^i$ in period 1.

Policy 2: $3 + 3\alpha + 3\alpha^2 + \cdots$

$$= \frac{3}{1-\alpha}$$

Poly 3 $= 2 + 4\alpha + 2\alpha^2 + 4\alpha^3 + \cdots$

$$= 2(1 + \alpha^2 + \alpha^4 + \cdots$$

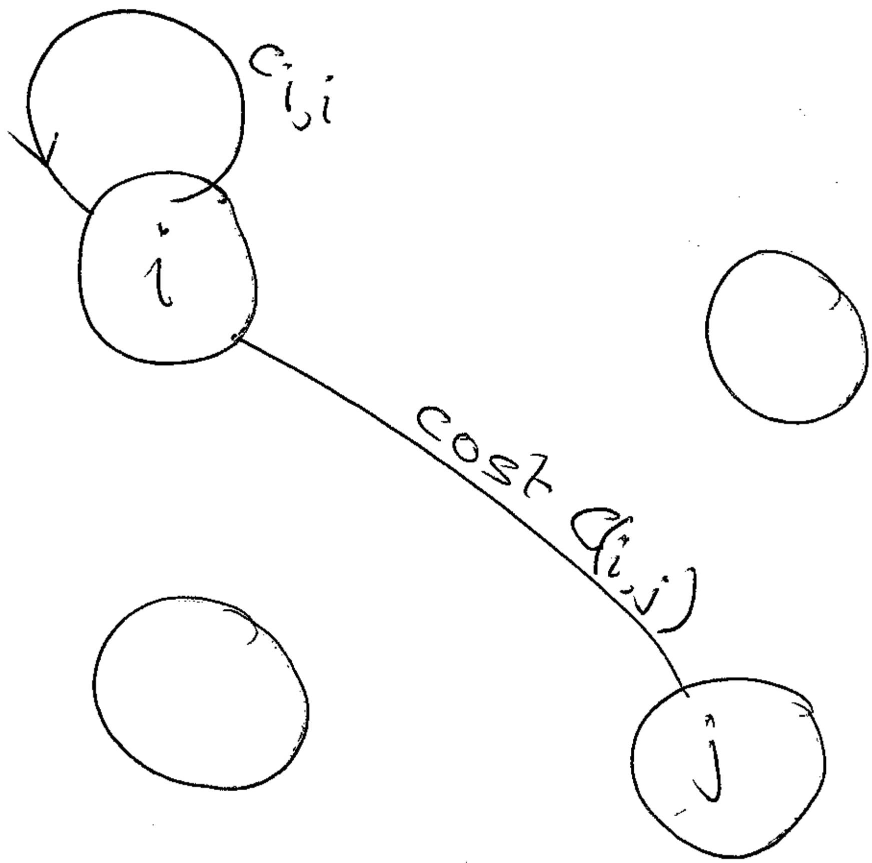
$$+ 4\alpha(1 + \alpha^2 + \alpha^4 + \cdots$$

$$= \frac{2 + 4\alpha}{1 - \alpha^2}$$

"System" consists of

n states . production problem

state = inventory



In state i, you choose which state to go to next.

If Process goes through states $i_1, i_2, \ldots$ then

$$\text{Cost} = c(i_1, i_2) + \alpha c(i_2, i_3) + \alpha^2 c(i_3, i_4) + \cdots$$

Problem: find sequence that minimise disounted cost of continuing forever.

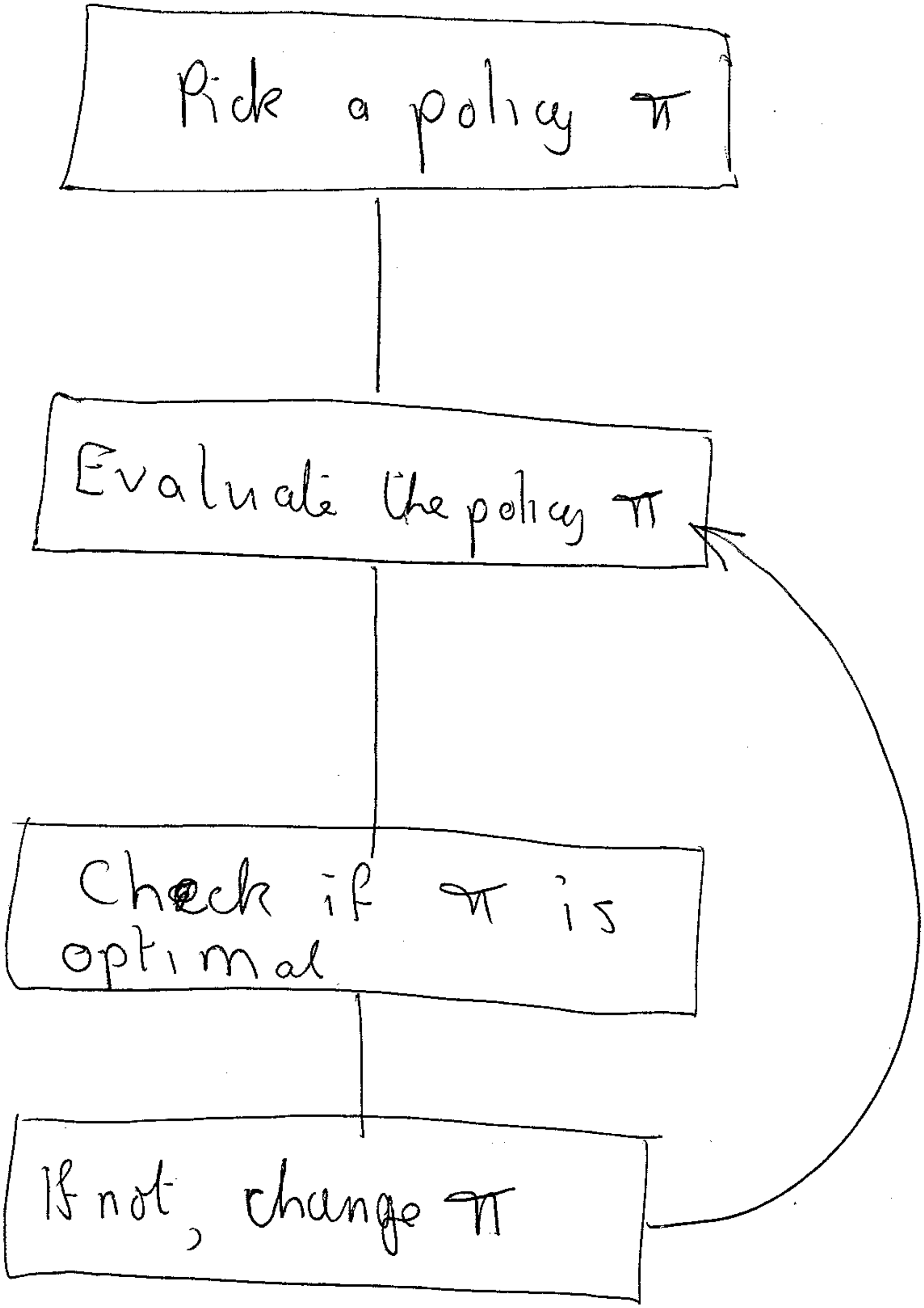
A <u>policy</u> is a function

$$\Pi : [n] \longrightarrow [n]$$

gives sequence

$$i,\ \pi(i),\ \pi^2(i),\ \pi^3(i),\ \ldots$$

Finding the best policy



Policy Evaluation

Suppose y_i = cost starting from i

$$= c(i, \pi(i)) + \alpha y_{\pi(i)}$$

n equations in n unknowns → solve.

$$(c(i,j)) = \begin{bmatrix} 3 & 1 & 4 & 5 \\ 2 & 2 & 1 & 6 \\ 7 & 1 & 3 & 4 \\ 1 & 4 & 2 & 5 \end{bmatrix}$$

$\alpha = \frac{1}{2}$

Start with

i	1	2	3	4
$\pi(i)$	4	4	1	4

If π ~~is~~ is optimal then

$$y_i = \min_j \left[c(i,j) + \alpha y_j \right] \quad (*)$$

I check (*).

If true — done

If $\neg$ true — change π

For each i change $\pi(i)$ to $\min_j$ in (*)

$$y_1 = 5 + \tfrac{1}{2} y_4 \quad = 10$$

$$y_2 = 6 + \tfrac{1}{2} y_4 \quad = 11$$

$$y_3 = 7 + \tfrac{1}{2} y_1 \quad = 12$$

$$y_4 = 5 + \tfrac{1}{2} y_4 \quad = 10$$

Policy Improvement