9\11\15

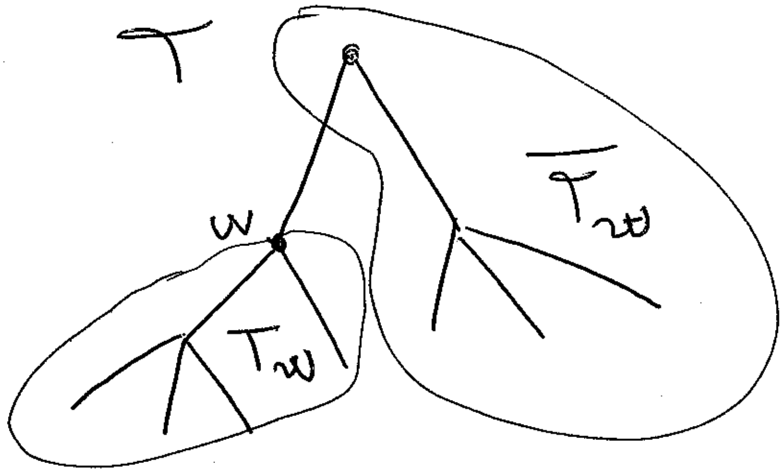
Stick problem: suppose we must make exactly k cuts.

$f_i(l)$ = max obtainable from $[0,l]$ with exactly i cuts

$$= \max_{i-1 \leq x < l} \left[v_{xl} + f_{i-1}(x) \right]$$

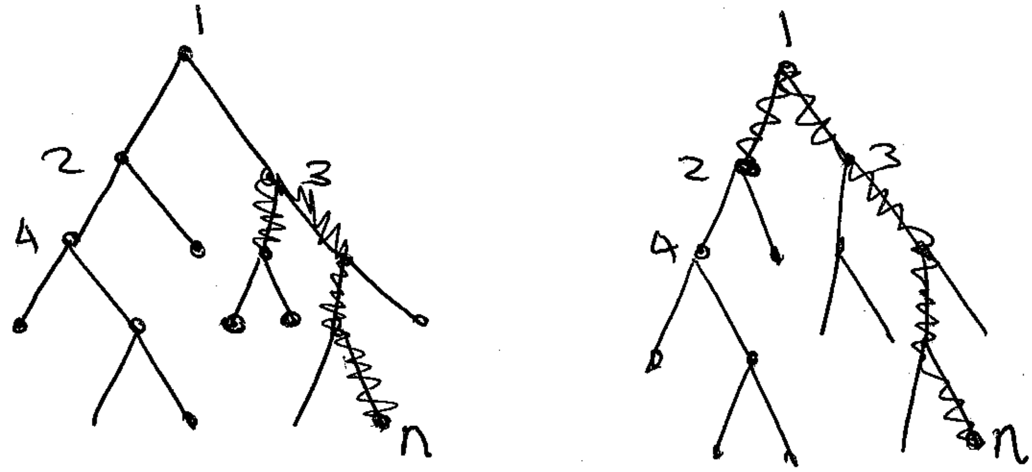
We want $f_k(L)$.

Suppose instead of cutting up a path, we want to cut up a tree:



Suppose that each path P in the tree has a value $v(P)$ and we want to partition vertices into vertex dijoint paths of maximum total value.

We let T_v be the subtree below vertex v and $\overline{T}_v = T \setminus T_v$.



$$f(T) = \max_{x} \Big[v(\text{path } P \text{ from } x \leftrightarrow n) \quad (*)$$
$$+ \sum_{\substack{\text{trees } X \\ \text{in forest } T \setminus P}} f(X) \Big]$$

All trees in X are of the form T_w or $\overline{T}_w$ for some $w \in T$. So there is an ordering of the trees so that we compute $f(X)$ before needed for $(*)$.

Adding some probability

Production problem. Suppose the demand is now random.

Assume that demand d in any period satisfies

$$\Pr[\text{demand} = d] = p_d, \quad d = 0, 1, 2 \ldots$$

in current period

Poss 1: know demand therefore decide level of production. Poss 2: after.

Goal 1: minimise expected cost of meeting all demand.
(Assumes Poss 1.

Goal 2: Maximise probability of satisfying 90% of demand in each period.

Goal 3. Minimise cost subject to prob. of meeting demand in each period $\geq 50\%$.

$f_r(i) =$ min
expected cost of meeting all
demand

$$= \sum_d p_d \left[\min_x \left[c(x) + f_{r+1}(x+i-d) \right] \right]$$